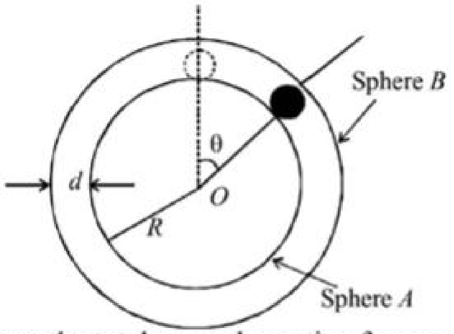
Q 6

A spherical ball of mass m is kept at the highest point in the space between two fixed, concentric spheres A and B (see figure). The smaller sphere A has a radius R and the space between the two spheres has a width d. The ball has a diameter very slightly less than d. All surfaces are frictionless. The ball is given a gentle push (towards the right in the figure). The angle made by the radius vector of the ball with the upward vertical is denoted by θ (shown in the figure). *(2002 - 5 Marks)*



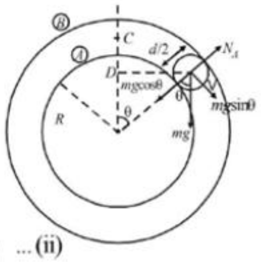
(a) Express the total normal reaction force exerted by the sphere on the ball as a function of angle θ.

(b) Let N_A and N_B denote the magnitudes of the normal reaction forces on the ball exerted by the sphere A and B, respectively. **Sketch** the variations of N_A and N_B as functions of $\cos\theta$ in the range $0 \le \theta \le \pi$ by drawing two **separate** graphs in your answer book, taking $\cos\theta$ on the horizontal axes.

Ans

The ball is moving in a circular motion. The necessary centripetal force is provided by $(mg\cos\theta - N)$. Therefore,

$$mg\sin\theta - N_A = \frac{mv^2}{\left(R+\frac{d}{2}\right)} \quad \text{... (i)}$$



According to energy conservation

$$\frac{1}{2}mv^2 = mg\left(R+\frac{d}{2}\right)(1-\cos\theta) \quad \text{... (ii)}$$

From (i) and (ii)

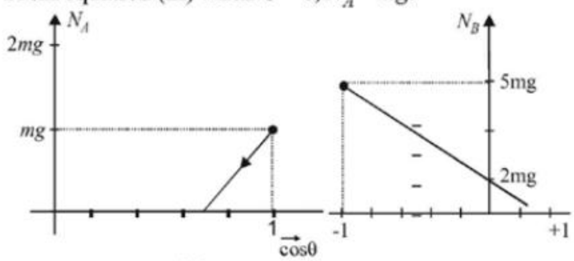
$$N_A = mg(3\cos\theta - 2) \quad \text{... (iii)}$$

The above equation shows that as θ increases N_A decreases. At a particular value of θ, N_A will become zero and the ball will lose contact with sphere A. This condition can be found by putting $N_A = 0$ in eq. (iii)

$$0 = mg(3\cos\theta - 2) \therefore \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

The graph between N_A and $\cos\theta$

From equation (iii) when $\theta = 0$, $N_A = mg$.



When $\theta = \cos^{-1}\left(\frac{2}{3}\right)$; $N_A = 0$

The graph is a straight line as shown.

when $\theta > \cos^{-1}\left(\frac{2}{3}\right)$; $N_B - (mg\cos\theta) = \frac{mv^2}{R+\frac{d}{2}}$

$$\Rightarrow N_B + mg\cos\theta = \frac{mv^2}{\left(R+\frac{d}{2}\right)} \quad \text{... (iv)}$$

Using energy conservation

$$\frac{1}{2}mv^2 = mg\left[\left(R+\frac{d}{2}\right)-\left(R+\frac{d}{2}\right)\cos\theta\right]$$

$$\frac{mv^2}{\left(R+\frac{d}{2}\right)} = 2mg\,[1-\cos\theta\,] \qquad \text{... (v)}$$

From (iv) and (v), we get

$$N_B + mg\cos\theta = 2mg - 2mg\cos\theta$$

$$N_B = mg\,(2-3\cos\theta\,)$$

When $\cos\theta = \frac{2}{3}, N_B = 0$

When $\cos\theta = -1, N_B = 5\,mg$

Therefore the graph is as shown.