14. The work done on a particle of mass m by a force,

$$K\left[\frac{x}{(x^2+y^2)^{3/2}}\hat{i}+\frac{y}{(x^2+y^2)^{3/2}}\hat{j}\right]$$

(K being a constant of appropriate dimensions), when the particle is taken from the point $(a, 0)$ to the point $(0, a)$ along a circular path of radius a about the origin in the $x-y$ plane is *(JEE Adv. 2013)*

(a) $\frac{2K\pi}{a}$ (b) $\frac{K\pi}{a}$ (c) $\frac{K\pi}{2a}$ (d) 0

Correct option is D)

Given that,

$$\text{Force } F = K\left[\frac{x}{(x^2+y^2)^{\frac{3}{2}}}\hat{i}+\frac{y}{(x^2+y^2)^{\frac{3}{2}}}\hat{j}\right]$$

Now, for the small distance is dr travelled by the particle in the direction

$d\vec{r} = d\vec{x}\hat{i} + d\vec{y}\hat{j}$

Now, small work done is

$dw = F.d\vec{r}$

The total work done

$W = \int dw$

$W = \int F \cdot d\vec{r}$

$$W = \int K\left[\frac{x}{(x^2+y^2)^{\frac{3}{2}}}\hat{i}+\frac{y}{(x^2+y^2)^{\frac{3}{2}}}\hat{j}\right]\cdot\left[dx\hat{i}+dy\hat{j}\right]$$

$$W = K\int_a^0\left[\frac{x}{(x^2+y^2)^{\frac{3}{2}}}\hat{i}\right]+\int_0^a\left[\frac{y}{(x^2+y^2)^{\frac{3}{2}}}\hat{j}\right]$$

Now, solve the first term of integration

Put,

$x^2+y^2=t$

$2xdx = dt$

Now,

$$=\frac{K}{2}\int_a^0\frac{2xdx}{(x^2+y^2)^{\frac{3}{2}}}$$

$$=\frac{K}{2}\int_a^0\frac{dt}{(t)^{\frac{3}{2}}}$$

$$=\frac{K}{2}\times 2a^{\frac{1}{2}}$$

$$=Ka^{\frac{1}{2}}$$

Now, similarly for second term of integration

$$=-Ka^{\frac{1}{2}}$$

Now, work done is

$$W = Ka^{\frac{1}{2}} - Ka^{\frac{1}{2}}$$

$W = 0$

Hence, the work done on a particle is 0