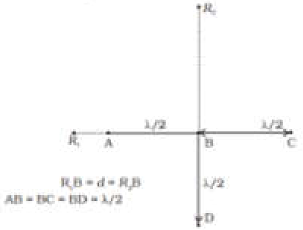
5. Four identical monochromatic sources A, B, C, D as shown in the (Figure) produce waves of the same wavelength λ and are coherent. Two receiver R_1 and R_2 are at great but equal distances from B.



a. Which of the two receivers picks up the larger signal?
b. Which of the two receivers picks up the larger signal when B is turned off?
c. Which of the two receivers picks up the larger signal when D is turned off?
d. Which of the two receivers can distinguish which of the sources B or D has been turned off?

Sol. Consider the disturbances at the receiver R_1 which is at a distance d from B.

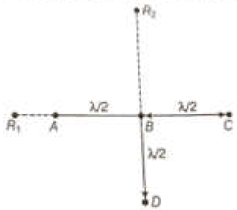
Let the wave at R_1 because of A be y_A $a\cos\omega t$. The path difference of the signal from A with from B is $\frac{\lambda}{2}$ and hence, the phase difference is π

Thus, the wave at R_1 because of B is

$y_B = a\cos(\omega t - \pi) = -a\cos\omega t$

The path difference of the signal from C with that from A is λ and hence the phase difference is 2π

Thus, the wave at R_1 because of C is $y_C = a\cos(\omega t - 2\pi) = a\cos\omega t$



The path difference between the signal from D with that of A is

$\sqrt{d^2 + \left(\frac{\lambda^2}{2}\right)} - (d - \frac{\lambda}{2}) = d(1 + \frac{\lambda}{4d^2})^{1/2} - d + \frac{\lambda}{2}$

$= d(1 + \frac{\lambda}{4d^2})^{1/2} - d + \frac{\lambda}{2} \approx \frac{\lambda}{2}\ (\therefore d >> \lambda)$

Therefore, the phase difference is π

$\therefore Y_D = a\cos(\omega t - \pi) = -a\cos\omega t$

Thus, the signal picked up at R_1 from all the four sources is $Y_{R_1} = y_A + y_B + y_C + y_D$

$= a\cos\omega t - a\cos\omega t + a\cos\omega t - a\cos\omega t = 0$

a. Let the signal picked up at R_2 from B be $Y_B = a_1\cos\omega t$

The path difference between signal at D and that at B is $\frac{\lambda}{2}$

$\because y_D = -a_1\cos\omega t$

The path difference between signal at A and that at B is

$\sqrt{d^2 + \left(\frac{\lambda^2}{2}\right)} - d = d\left(1 + \frac{\lambda^2}{4d^2}\right)^{1/2} - d \quad \sim \frac{1\lambda^2}{8d^2}$

As d >> λ, therefore this path difference $\to 0$

and phase difference $= \frac{2\pi}{\lambda}\left(\frac{1}{8}\frac{\lambda^2}{d^2}\right) \to 0$

Hence, $y_A = a_1\cos(\omega t - \phi)$

Similarly, $y_C = a_1\cos(\omega t - \phi)$

$\therefore$ Signal picked up by R_2 is

$y_A + y_B + y_C + y_D = y = 2a_1\cos(\omega t - \phi)$

$\therefore y|^2 = 4a_1^2\cos^2(\omega t - \phi)$

$\therefore < I >= 2a_1^2$

Thus, R_1 picks up the larger isgnal.

b. If B is switched off,

R_1 picks up y = $a\cos\omega t$

$\langle I_{R_1}\rangle = \frac{1}{2}a^2$

R_2 pick up $y = a\cos\omega t$

$\therefore \langle I_{R_2}\rangle = a^2 < \cos^2\omega t > y = \frac{a^2}{2}$

c. Thus, R_1 and R_2 pick up the same signal

If D is switched off.

R_1 pick up $y = a\cos\omega t$

$\therefore \langle I_{R_1}\rangle = \frac{1}{2}a^2$

R_2 picks up $y = a\cos\omega t$

$\therefore \langle I_{R_2}\rangle = 9a^2 < \cos^2\omega t > y = \frac{9a^2}{2}$

Thus, R_2 picks up larger signal compared to R_1.

d. Thus, a signal at R1 indicates B has been switched off and an enhanced signal at R_2 indicates D has been switched off.