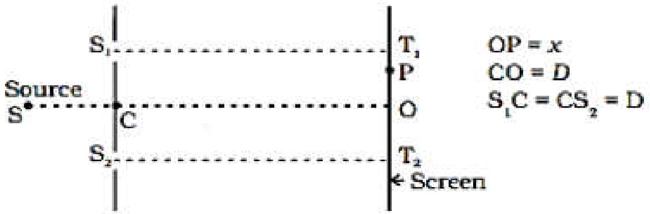
3. Consider a two-slit interference arrangements (Figure) such that the distance of the screen from the slits is half the distance between the slits. Obtain the value of D in terms of λ such that the first minim on the screen fall at a distance D from the center O.



Sol. According to θ

d = D (Given) ...(i)

$D = \frac{1}{2}d$ (Given) ...(ii)

d = 2D

Path difference at P = $S_2P - S_1P$

Path difference $p = \sqrt{D^2 + \left(x + \frac{d}{2}\right)^2} - \sqrt{D^2 + \left(x - \frac{d}{2}\right)^2}$

Substitute the value of d and x from (i) and (ii)

$= \sqrt{D^2 + (D) + D)^2} - \sqrt{D^2 + (D - D)^2}$

$= \sqrt{5D^2} - \sqrt{D^2}$

$p = D(\sqrt{5} - 1)$

The path difference for nth dark fringe from central maxima O is $(2n-1)\frac{\lambda}{2}$

$\therefore$ For 1st minima $p = \frac{\lambda}{2}$

Put the value of p in (iii)

$\frac{\lambda}{2} = D(\sqrt{5} - 1)$

$D = \frac{\lambda}{2(\sqrt{5}-1)}$

Rationalizing the denominator, we get,

$D = \frac{\lambda}{2(\sqrt{5}-1)} \times \frac{(\sqrt{5}+1)}{(\sqrt{5}+1)} = \frac{(2.236+1)}{2\times(5-1)}\lambda = \frac{3.236}{2\times4}\lambda$

$= \frac{3.236}{8}\lambda = 0.404\lambda$