For given value of u, there are two angles of projection for which the horizontal range is the same. Show that the sum of the maximum heights for these two angles is independent of the angle of projection.

Solution There are two angles of projection α and $90° - \alpha$ for which the horizontal range R is same.

Now,
$$H_1 = \frac{u^2 \sin^2 \alpha}{2g} \quad \text{and} \quad H_2 = \frac{u^2 \sin^2 (90° - \alpha)}{2g} = \frac{u^2 \cos^2 \alpha}{2g}$$

Therefore,
$$H_1 + H_2 = \frac{u^2}{2g} (\sin^2 \alpha + \cos^2 \alpha)$$

$$= \frac{u^2}{2g}$$

Clearly the sum of the heights for the two angles of projection is independent of the angles of projection.