QUES 02:-

The trajectory of a projectile in a vertical plane is $y = \alpha x - \beta x^2$, where α and β are constants and x & y are respectively the horizontal and vertical distances of the projectile from the point of projection. The angle of projection θ and the maximum height attained H are respectively given by [Feb. 26, 2021 (II)]

(a)
$$\tan^{-1} \alpha, \frac{4\alpha^2}{\beta}$$
 (b) $\tan^{-1} \beta, \frac{\alpha^2}{2\beta}$

(b)
$$\tan^{-1}\beta, \frac{\alpha^2}{2\beta}$$

(c)
$$\tan^{-1}\left(\frac{\beta}{\alpha}\right), \frac{\alpha^2}{\beta}$$
 (d) $\tan^{-1}\alpha, \frac{\alpha^2}{4\beta}$

(d)
$$\tan^{-1} \alpha, \frac{\alpha^2}{4\beta}$$

(d) Given:
$$y = \alpha x - \beta x^2$$

For maximum height, maximum value of y

$$\frac{dy}{dx} = 0 \Rightarrow \frac{d(\alpha x - \beta x^2)}{dx} = 0 \Rightarrow \alpha - 2\beta x = 0$$

$$\therefore x = \frac{\alpha}{2\beta}$$

$$\therefore y = \alpha \left(\frac{\alpha}{2\beta}\right) - \beta \left(\frac{\alpha^2}{4\beta^2}\right)$$

$$\Rightarrow \left(\frac{\alpha^2}{2\beta}\right) - \left(\frac{\alpha^2}{4\beta}\right) \Rightarrow \frac{\alpha^2}{4\beta}$$

So,
$$H_{\text{max}} = \frac{\alpha^2}{4\beta}$$

Hence option (d) is correct answer.