Question 05. *A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β, to come to rest. If the total time elapsed is t seconds, then evaluate (a) the maximum velocity reached and (b) the total distance travelled.*

Solution (a) Let the car accelerates for time t_1 and decelerates for time t_2. Then,

$$t = t_1 + t_2 \qquad \ldots(i)$$

and corresponding velocity-time graph will be as shown in Fig. 6.32.

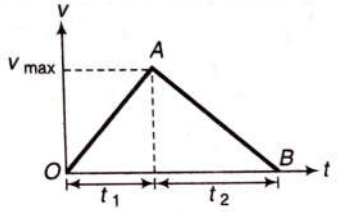

Fig. 6.32

From the graph,

$$\alpha = \text{slope of line } OA = \frac{v_{max}}{t_1} \quad \text{or} \quad t_1 = \frac{v_{max}}{\alpha} \qquad \ldots(ii)$$

and $$\beta = -\ \text{slope of line } AB = \frac{v_{max}}{t_2}$$

or $$t_2 = \frac{v_{max}}{\beta} \qquad \ldots(iii)$$

From Eqs. (i), (ii) and (iii), we get

$$\frac{v_{max}}{\alpha} + \frac{v_{max}}{\beta} = t$$

or $$v_{max}\left(\frac{\alpha+\beta}{\alpha\beta}\right) = t$$

or $$v_{max} = \frac{\alpha\beta t}{\alpha+\beta} \qquad \textbf{Ans.}$$

(b) Total distance = total displacement = area under v-t graph

$$= \frac{1}{2} \times t \times v_{max}$$

$$= \frac{1}{2} \times t \times \frac{\alpha\beta t}{\alpha+\beta}$$

or $$\text{Distance} = \frac{1}{2}\left(\frac{\alpha\beta t^2}{\alpha+\beta}\right) \qquad \textbf{Ans.}$$