

Q 1.

The distance  $x$  covered by a particle in one dimensional motion varies with time  $t$  as  $x^2 = at^2 + 2bt + c$ . If the acceleration of the particle depends on  $x$  as  $x^{-n}$ , where  $n$  is an integer, the value of  $n$  is \_\_\_\_\_.

[Main 9 Jan 2020 (I)]

ans

$$\begin{aligned} & \text{Distance } X \text{ varies with time } t \text{ as } x^2 = at^2 + 2bt + c \\ \Rightarrow & 2x \frac{dx}{dt} = 2at + 2b \Rightarrow x \frac{dx}{dt} = at + b \Rightarrow \frac{dx}{dt} = \frac{(at + b)}{x} \\ \Rightarrow & x \frac{d^2x}{dt^2} + \left( \frac{dx}{dt} \right)^2 = a \\ \Rightarrow & \frac{d^2x}{dt^2} = \frac{a - \left( \frac{dx}{dt} \right)^2}{x} = \frac{a - \left( \frac{at + b}{x} \right)^2}{x} \\ = & \frac{ax^2 - (at + b)^2}{x^3} = \frac{ac - b^2}{x^3} \\ \Rightarrow & a \propto x^{-3} \quad \text{Hence, } n = 3 \end{aligned}$$