Q 3. Consider a circular current-carrying loop of radius R in the x-y plane with centre at origin. Consider the line intergral $\Im$ (L) = $\left|\int_{-L}^{L} \mathbf{B}\cdot\mathbf{dl}\right|$ taken along z-axis.

a. Show that $\Im$ (L) monotonically increases with L.
b. Use an appropriate Amperian loop to show that $\Im(\infty) = \mu_0 I$, where I is the current in the wire.
c. Verify directly the above result.
d. Suppose we replace the circular coil by a square coil of sides R carrying the same current I. What can you say about $\Im$(L) and $\Im(\infty)$?

Sol.

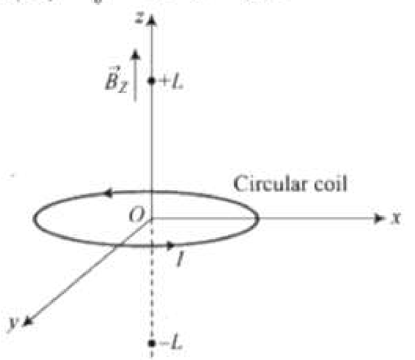
a. Magnetic field due to a circular current-carrying loop lying in the xy- plane acts along z-axis as shown in figure.

$$\Im(L) = \left|\int_{-L}^{L} \vec{B}\cdot\vec{dl}\right| = \int_{-L}^{+L} B dl \cos 0^\circ = \int_{-L}^{+L} B dl = 2BL$$

$\therefore \Im$ (L) is monotonically increasing function of L.

b. Now consider an Amperean loop around the circular coil of such a large radius $L \to \infty$. since this loop encloses a current I, Now using Ampere's law

$$\Im(\infty) = \oint^{+\infty} \vec{B}\cdot\vec{dl} = \mu_0 I$$



c. The magnetic field at the axis (z-axis) of circular coil at a distance z from the centre of a circular coil of radius R carrying current I is given by $B_z = \frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}}$

Now integrating

$$\int_{-\infty}^{+\infty} B_z dz = \int_{-\infty}^{+\infty} \frac{\mu_0 I R^2}{2(z^2+R^2)^{3/2}} dz$$

Let z = R tan θ so that dz = R sec$^2\theta$ dθ

and $(z^2 + R^2)^{3/2} = (R^2 \tan^2\theta + R^2)^{3/2}$

$= R^3 \sec^3\theta$ (as $1 + \tan^2\theta = \sec^2\theta$)

Thus, $\int_{-\infty}^{+\infty} B_z dz = \frac{\mu_0 I}{2}\int_{-\pi/2}^{+\pi/2} \frac{R^2 (R\sec^2\theta d\theta)}{R^3\sec^3\theta}$

$= \frac{\mu_0 I}{2}\int_{-\pi/2}^{+\pi/2} \cos\theta d\theta = \mu_0 I$

d. As we know $(B_Z)_{square} < (B_z)_{circular\ coil}$

For the same current, and side of the square equal to radius of the coil

$\Im(\infty)_{square} < \Im(\infty)_{circularcoil}$

By using the same argument as we done in case (b), it can be shown that

$\Im(\infty)_{square} = \Im(\infty)_{circularcoil} = \mu_0 I$