

2. Two vectors  $\vec{X}$  and  $\vec{Y}$  have equal magnitude. The magnitude of  $\vec{X} - \vec{Y}$  is  $n$  times the magnitude of  $(\vec{X} + \vec{Y})$ .

The angle between  $\vec{X}$  and  $\vec{Y}$  is : [July 25, 20, 2021 (II)]

(a)  $\cos^{-1}\left(\frac{1-n^2}{1+n^2}\right)$       (b)  $\cos^{-1}\left(\frac{n^2-1}{-n^2-1}\right)$

(c)  $\cos^{-1}\left(\frac{n^2+1}{-n^2-1}\right)$       (d)  $\cos^{-1}\left(\frac{n^2+1}{n^2-1}\right)$

**ans** (a) We have given,

$$|\vec{X} - \vec{Y}| = n|\vec{X} + \vec{Y}|$$

$$\text{As } |\vec{A} + \vec{B}|^2 = |\vec{A}|^2 + |\vec{B}|^2 + 2|\vec{A}||\vec{B}|\cos\theta$$

$$\text{and } |\vec{A} - \vec{B}|^2 = |\vec{A}|^2 + |\vec{B}|^2 - 2|\vec{A}||\vec{B}|\cos\theta$$

$$\therefore |\vec{X}|^2 + |\vec{Y}|^2 - 2|\vec{X}||\vec{Y}|\cos\theta$$

$$= n^2 \left[ |\vec{X}|^2 + |\vec{Y}|^2 + 2|\vec{X}||\vec{Y}|\cos\theta \right]$$

$$\text{Given, } |\vec{X}| = |\vec{Y}|$$

$$\Rightarrow 2|\vec{X}|^2 - 2|\vec{X}|^2 \cos\theta = 2n^2|\vec{X}|^2 + 2n^2|\vec{X}|^2 \cos\theta$$

$$\Rightarrow 1 - \cos\theta = n^2 + n^2 \cos\theta$$

$$\Rightarrow \cos\theta = \frac{1-n^2}{1+n^2}$$

$$\Rightarrow \theta = \cos^{-1} \frac{1-n^2}{1+n^2}$$