

5. Three vectors A, B and C add up to zero. Find which is false.

- i. $(A \times B) \times C$ is not zero unless B, C are parallel
- ii. $(A \times B) \cdot C$ is not zero unless B, C are parallel
- iii. If A, B, C define a plane, $(A \times B) \times C$ is in that plane
- iv. $(A \times B) \cdot C = |A| |B| |C| \rightarrow C^2 = A^2 + B^2$

Sol.

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We have to identify a false statement from above

We have given that $\vec{A} + \vec{B} + \vec{C} = 0$

Therefore taking the cross product on both sides

$$\vec{A} + \vec{B} + \vec{C} = 0$$

$$\vec{B} \times (\vec{A} + \vec{B} + \vec{C}) = \vec{B} \times \vec{0}$$

$$\vec{B} \times \vec{A} + \vec{B} \times \vec{B} + \vec{B} \times \vec{C} = \vec{0}$$

Now we know that when vectors are parallel then their cross product is zero. $\vec{B} \times \vec{B} = 0$

$$\vec{A} \times \vec{B} = \vec{B} \times \vec{C}$$

Taking post Cross Product on both sides with c

$$(\vec{A} \times \vec{B}) \times \vec{C} = (\vec{B} \times \vec{C}) \times \vec{C}$$

Now this could only zero when B and C are parallel to each other as

$$\vec{B} \times \vec{C} = |B| |C| \sin \theta = 0 \text{ only when } \theta = 0 \text{ that's when B and C are parallel}$$

Therefore statement A is true.

Now taking the previous equation

$$\vec{A} \times \vec{B} = \vec{B} \times \vec{C}$$

Taking dot product with c on both sides

$$(\vec{A} \times \vec{B}) \cdot \vec{C} = (\vec{B} \times \vec{C}) \cdot \vec{C}$$

Now this could be zero on two conditions first is that B and C are parallel but it could be zero without C being parallel to B. As when we will take the cross product of B and C, then vector perpendicular to both B and C, say vector K. And by taking the dot product of K and C it will also be zero as the angle between them will always be 90.

Therefore B is false

Now if vector triple product of A and B and C, then vector will always lie on the plane which will be formed by A, B and C. This could be visualized by understanding that $\vec{A} + \vec{B} + \vec{C} = 0$ will always lie in a single plane forming sides of the triangle.

Now,

$$\vec{A} \times \vec{B} = \vec{K}$$

K will be perpendicular to the plane containing A and B.

And taking the cross product with C (which is also lying on the same plane as that of A and B) will give a vector which is perpendicular to C but will be lying on the same plane as that of A, B and C. Therefore statement C is true.

It is given in last option that $|\vec{A} \times \vec{B}| = |A| |B|$, therefore, the angle between vector A and vector B is 90 and we know that

$\vec{A} + \vec{B} + \vec{C} = 0$, therefore $|A|, |B|, |C|$ form a triangle with the angle between A and B equal to 90, therefore, it is a right-angled triangle.

Hence option D is also true.