Related Questions with Solutions

Questions

Quetion: 01

Given that $(1 + x + x^2)^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$, Let
$a_0 + a_1 + a_2 + \ldots + a_{2n} = \alpha$
$a_0 - a_1 + a_2 - a_3 \ldots + a_{2n} = \beta$
$a_0^2 - a_1^2 + a_2^2 - a_3^2 + \ldots + a_{2n}^2 = \gamma$ then $\alpha + \beta + \gamma =$

A. 0
B. $3^n + 1 + a_n$
C. $3^n + 1 - a_n$
D. $3^n - 1 + a_n$

Solutions

Solution: 01

$[1 + x + x^2]^n = a_0 + a_1x + a_2x^2 + ... + a_{2n}x^{2n}$
put x = 1,
$3^n = a_0 + a_1 + a_2 + ... + a_{2n} = \alpha$
put x = – 1,
$1 = a_0 - a_1 + a_2 ... + a_{2n} = \beta$

$$\left(1+x+x^2\right)^n\left(1-\frac{1}{x}+\frac{1}{x^2}\right)^n =$$

$$\left(\alpha_0+a_ix+\ldots+a_{2n}x^{2n}\right)\left(a_0-\frac{a_1}{x}+\frac{a_2}{x^2}\ldots+\frac{a_{2n}}{x^{2n}}\right)$$

coefficient of x^0 in $\left(1+x+x^2\right)^n\left(1-\frac{1}{x}+\frac{1}{x^2}\right)$ is

$a_0^2 - a_1^2 + a_2^2 \ldots + o_{2n}^2$

coefficient of x^0 in $\left(1+x+x^2\right)^n\left[1-\frac{1}{x}+\frac{1}{x^2}\right]^n$

$$\frac{\left(x^2+x+1\right)^n\left(x^2-x+1\right)^n}{x^{2n}}$$

$$\frac{\left(x^4+x^2+1\right)^n}{x^{2n}}$$

coefficient of x^{2n} in $[1 + x^2 + x^4]^n$

let $x^2 = t$, $[1 + t + t^2]^n = a_0 + a_1t + ... + a_{2n}t^n$

So, coefficient of t^n is a_n

Hence, coefficient of x^{2n} is $a_n = \gamma$

$\therefore \alpha + \beta + \gamma = 3^n + 1 + a_n$

Correct Options

Answer:01
Correct Options: B