Concepts and Formulas

Infinite Binomial Expansions: when n is rational

General Form: for any x less than 1,

$$(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots +$$
$$+\frac{n(n-1)(n-2)\ldots(n-r+1)}{r!}x^r + \ldots\infty$$

Important Expansion:

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \ldots$$

$$e^{-x} = 1 - x + \frac{x^2}{2} - \frac{x^3}{3!} + \frac{x^4}{4!} - \frac{x^5}{5!} \ldots\ldots$$

Problem Types:

1. When three consecutive terms are given to be and Arithmetic Progression(AP):

   Result used to solve:

   first term + third term = 2*(second term)

2. Finding various sums of binomial coefficients:

   Here trick is to be able to figure out on which expression to use binomial theorem

3. Conditioning on binomial terms:

   1. first write down the condition that is given on terms
   2. then simply follow solving steps to get answer

Examples for these types of questions from NCERT:

1.

Example 11 If the coefficients of a^{r-1}, a^r and a^{r+1} in the expansion of $(1 + a)^n$ are in arithmetic progression, prove that $n^2 - n(4r + 1) + 4r^2 - 2 = 0$.

2.

Example 18 If $(1 - x + x^2)^n = a_0 + a_1 x + a_2 x^2 + ... + a_{2n} x^{2n}$, then $a_0 + a_2 + a_4 + ... + a_{2n}$ equals.

(A) $\dfrac{3^n + 1}{2}$ (B) $\dfrac{3^n - 1}{2}$ (C) $\dfrac{1 - 3^n}{2}$ (D) $3^n + \dfrac{1}{2}$

3.

Example 17 If the coefficients of $(r - 5)^{th}$ and $(2r - 1)^{th}$ terms in the expansion of $(1 + x)^{34}$ are equal, find r.

Watch video and try to solve them yourself, since these are ncert examples their answer is given in the ncert book.