Binomial Theorem - Class XI

Past Year JEE Questions

Questions

Quetion: 01

Let $S_n = 1 + q + q^2 + \ldots\ldots + q^n$ and $T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + \ldots\ldots + \left(\frac{q+1}{2}\right)^n$ where q is a real number and $q \neq 1$. If $^{101}C_1 + {}^{101}C_2 \cdot S_1 + \ldots + {}^{101}C_{101} \cdot S_{100} = \alpha T_{100}$ then α is equal to

A. 202

B. 200

C. 2^{100}

D. 2^{99}

Solutions

Solution: 01

Explanation

$^{101}C_1 + {}^{101}C_2 S_1 + \ldots\ldots + {}^{101}C_{101} S_{100}$

$= \alpha T_{100}$

$^{101}C_1 + {}^{101}C_2(1 + q) + {}^{101}C_3(1 + q + q^2) +$

$\ldots\ldots + {}^{101}C_{101}(1 + q + \ldots\ldots + q^{100})$

$= 2\alpha \frac{\left(1 - \left(\frac{1+q}{2}\right)^{101}\right)}{(1-q)}$

$\Rightarrow {}^{101}C_1(1 - q) + {}^{101}C_2(1 - q^2) +$

$\ldots\ldots + {}^{101}C_{101}(1 - q^{101})$

$= 2\alpha \left(1 - \left(\frac{1+q}{2}\right)^{101}\right)$

$\Rightarrow (2^{101} - 1) - ((1 + q)^{101} - 1)$

$= 2\alpha \left(1 - \left(\frac{1+q}{2}\right)^{101}\right)$

$\Rightarrow 2^{101}\left(1 - \left(\frac{1+q}{2}\right)^{101}\right) = 2\alpha \left(1 - \left(\frac{1+q}{2}\right)^{101}\right)$

$\Rightarrow \alpha = 2^{100}$