Binomial Theorem - Class XI

Past Year JEE Questions

Questions

Quetion: 01

Let $(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + ... + a_{40}x^{40}$. Then $a_1 + a_3 + a_5 + ... + a_{37}$ is equal to

A. $2^{20}(2^{20} - 21)$

B. $2^{19}(2^{20} - 21)$

C. $2^{19}(2^{20} + 21)$

D. $2^{20}(2^{20} + 21)$

Solutions

Solution: 01

Explanation

$(1 + x + 2x^2)^{20} = a_0 + a_1x + a_2x^2 + ... + a_{40}x^{40}$

Put x = 1

$\Rightarrow 4^{20} = a_0 + a_1 + ... + a_{40}$..... (i)

Put x = −1

$\Rightarrow 2^{20} = a_0 - a_1 + ... + - a_{39} + a_{40}$..... (ii)

by (i) − (ii) we get,

$4^{20} - 2^{20} = 2(a_1 + a_3 + ... + a_{37} + a_{39})$

$\Rightarrow a_1 + a_3 + ... + a_{37} = 2^{39} - 2^{19} - a_{39}$..... (iii)

a_{39} = coeff. x^{39} in $(1 + x + 2x^2)^{20}$

$= \frac{20!}{0!19!}(1)^0(1)^1(2)^{19}$

$= 20.2^{19}$

$\therefore a_1 + a_3 + ... + a_{37} = 2^{39} - 2^{19}.21$

$\Rightarrow 2^{19}(2^{20} - 21)$