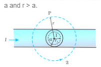
QUES 02:-

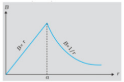
Figure shows a long straight wire of a circular cross-section (radius a) carrying steady current I. The current I is uniformly distributed across this cross-section. Calculate the magnetic field in the region r < a and r > a.



Sol.

a. Figure shows a long straight wire of a circular cross-section (radius a) carrying steady current I. The current I is uniformly distributed across this cross-section. Consider the case r > a. The Amperian loop, labelled 2, is a circle concentric with the cross-section. For this loop,
   $L = 2\pi r$
   I_e = Current enclosed by the loop = I
   The result is the familiar expression for a long straight wire
   $B(2\pi r) = \mu_0 I$
   $B = \frac{\mu_0 I}{2\pi r}$
   $B \propto \frac{1}{r} \quad (r > a)$

b. Consider the case r < a. The Amperian loop is a circle labelled 1. For this loop, taking th radius of the circle to be r, $L = 2\pi r$ Now the current enclosed Ie is not I, but is less than this value. Since the current distribution is uniform, the current enclosed is, $I_e = I\left(\frac{\pi r^2}{\pi a^2}\right) = \frac{Ir^2}{a^2}$
   Using Ampere's law, $B(2\pi r) = \mu_0 \frac{Ir^2}{a^2}$
   $B = \left(\frac{\mu_0 I}{2a^2}\right) r$
   $B \propto r \quad (r < a)$



The figure shows a plot of the magnitude of B with distance r from the centre of the wire. The direction of the field is tangential to the respective circular loop (1 or 2) and given by the right-hand rule described earlier in this section. This example possesses the required symmetry so that Ampere's law can be applied readily.