QUES 04:-

A coil having N turns is wound tightly in the form of a spiral with inner and outer radii a and b respectively. When a current I passes through the coil, the magnetic field at the center is [2001S]

(a) $\frac{\mu_0 NI}{b}$ (b) $\frac{2\mu_0 NI}{a}$

(c) $\frac{2\mu_0 NI}{2(b-a)} \ln \frac{b}{a}$ (d) $\frac{2\mu_0 NI}{2(b-a)} \ln \frac{a}{b}$

(c) Let us consider an element of thickness dx of wire. Let it be at a distance x from the centre O.

Number of turns per unit length $= \frac{N}{b-a}$

$\therefore$ Number of turns in thickness $dx = \frac{N}{b-a} dx$

Magnetic field due to this small element at O

$$dB = \frac{\mu_0}{2} \frac{NI}{(b-a)} \frac{dx}{x}$$

$$B = \int_a^b \frac{\mu_0}{2} \frac{NI}{b-a} \frac{dx}{x} = \frac{\mu_0}{2} \frac{NI}{(b-a)}$$

$$\int_a^b \frac{dx}{x} = \frac{\mu_0}{2} \frac{NI}{(b-a)} (\log_e x)_a^b$$

$$\therefore \quad B = \frac{\mu_0}{2} \frac{NI}{(b-a)} \ln \frac{b}{a}$$