

QUES 07

For the one-dimensional motion, described by $x = t - \sin t$.

- (a) $v(t) > 0$ for all $t > 0$ (b) $v(t) < 0$ for all $t > 0$
(c) $a(t) > 0$ for all $t > 0$ (d) (b) lies between 0 and 2

Sol: (a, c) Position of the particle is given as a function of time i.e. $x = t - \sin t$. By differentiating this equation w.r.t. time we get velocity of the particle as a function of time.

$$\text{Velocity } v = \frac{dx}{dt} = \frac{d}{dt}(t - \sin t) = 1 - \cos t$$

If we again differentiate this equation w.r.t. time we will get acceleration of the particle as a function of time.

$$\text{Acceleration } a = \frac{dv}{dt} = \frac{d}{dt}(1 - \cos t) = \sin t$$

As acceleration $a > 0$ for all $t > 0$

Hence, $v(t) > 0$ for all $t > 0$

Velocity $v = 1 - \cos t$

When, $\cos t = 1$, velocity $v = 0$

$$v_{\text{min}} = 1 - (\cos 0)_{\text{min}} = 1 - (1) = 0$$

$$v_{\text{max}} = 1 - (\cos 2\pi)_{\text{min}} = 1 - (-1) = 2$$

Hence, v lies between 0 and 2.

Acceleration $a = \frac{dv}{dt} = \sin t$

When $t = 0$, $x = 0$, $v = 0$, $a = 0$

When $t = \frac{\pi}{2}$, $x = \text{positive}$, $v = 0$, $a = -1$ (negative)

When $t = \pi$, $x = \text{positive}$, $v = \text{positive}$, $a = 0$

When $t = 2\pi$, $x = 0$, $v = 0$, $a = 0$