

QUES 06

Two masses 8 kg and 12 kg are connected at the two ends of a light, inextensible string that goes over a frictionless pulley. Find the acceleration of the masses and the tension in the string when the masses are released.

Sol. The given system of two masses 8 kg, 12 kg and a pulley can be represented as shown in the following figure. In this case, weights of the two masses act downwards and tension T in the two cases act upwards:



Smaller mass is given by, $m_1 = 8$ kg

Larger mass is given by, $m_2 = 12$ kg

Tension in the string is given by = T

When released, mass m_2 , owing to its weight, moves downward with acceleration a , and mass m_1 moves upward with the same acceleration (as they constitute a single system). Hence,

By applying Newton's second law of motion to the system of masses:

For mass m_1 :

The equation of motion can be written as given below :

$T - m_1g = ma \dots (i)$ (as the tension T exceeding the weight m_1g causes the motion of mass m_1)

For mass m_2 :

The equation of motion can be written as given below:

$m_2g - T = m_2a \dots (ii)$ (here weight m_2g exceeding the tension T causes motion of mass m_2)

Adding equations (i) and (ii), we get:

$$(m_2 - m_1)g = (m_1 + m_2)a$$

$$\therefore a = \left(\frac{m_2 - m_1}{m_1 + m_2} \right) g \dots (iii)$$

$$= \left(\frac{12 - 8}{12 + 8} \right) \times 10 = \frac{4}{20} \times 10 = 2 \text{ m/s}^2$$

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Thus, the acceleration of both masses is 2 m/s^2 .

Substituting the value of 'a' in equation (ii), we get:

$$m_2g - T = m_2 \left(\frac{m_2 - m_1}{m_1 + m_2} \right) g$$

$$T = \left(m_2 - \frac{m_2^2 - m_1m_2}{m_1 + m_2} \right) g$$

$$= \left(\frac{2m_1m_2}{m_1 + m_2} \right) g$$

$$= \left(\frac{2 \times 12 \times 8}{12 + 8} \right) \times 10$$

$$= \frac{2 \times 12 \times 8}{20} \times 10 = 96 \text{ N}$$

Therefore, the tension in the string is given by 96 N.