QUES 03

. A rectangular box lies on a rough inclined surface. The coefficient of friction between the surface and the box is μ. Let the mass of the box be m.

i. At what angle of inclination θ of the plane to the horizontal will the box just start to slide down the plane?
ii. What is the force acting on the box down the plane, if the angle of inclination of the plane is increased to $\alpha > \theta$?
iii. What is the force needed to be applied upwards along the plane to make the box either remain stationary or just move up with uniform speed?
iv. What is the force needed to be applied upwards along the plane to make the box move up the plane with acceleration a?

Sol.

i. For this condition to be true, the forces must balance each other. The frictional force(f) is given by:
   $f = \mu N = \mu$ mg cos θ
   Therefore,
   mg sin θ - μ mg cos θ = 0
   $\Rightarrow \sin\theta = \mu\cos\theta$
   $\Rightarrow \theta = \tan^{-1}(\mu)$
   where, μ is the coefficient of friction, N is normal to the surface, g is the acceleration due to gravity and θ is the angle of the incline.
   So for θ equal to $\tan^{-1}(\mu)$, the box just starts to slide
ii. From the figure, we can resolve the forces for $\alpha > \theta$
   F = mg sin α - μ mg cos α
iii. In this scenario, as we trying to move the box up the plane, the direction of friction changes and is now acting down the plane. The equation of force is:
   F = mg sin α + μ mg cos α
iv. Here, the acceleration is in the direction of the force. Thus resolving the forces, we have,
   F_t = F + ma = mg sin α + μ mg cos α + ma