QUES 02

When a body slides down from rest along a smooth inclined plane making an angle of 45° with the horizontal, it takes time T to reach the bottom. When the same body slides down from rest along a rough inclined plane making the same angle and through the same distance, it is seen to take time pT, where p is some number greater than 1. Calculate the co-efficient of friction between the body and the rough plane.

Sol. As the body slides down from rest along a smooth plane inclined at angle 45^0 in Time T,

Initial velocity(u) = 0, distance along the inclined plane = s, time taken(t) = T

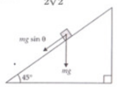
$a = g\sin 45^\circ = \frac{g}{\sqrt{2}}$, the component of acceleration along the inclined plane.

Now from Newton's equation of motion,

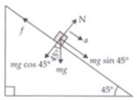
$s = ut + \frac{1}{2}at^2$

$\Rightarrow s = 0 + \frac{1}{2}\frac{g}{\sqrt{2}}T^2$

$\therefore s = \frac{gT^2}{2\sqrt{2}}$



Now for motion of the body along rough inclined plane, mg sin 45° exceeding static frictional motion μmg cos 45° will cause the downward motion of the body along the inclined plane with acceleration a.



Initial velocity, u = 0, distance along the inclined plane, $s = \frac{gT^2}{2\sqrt{2}}$ ....(i)

$\Rightarrow$ ma = mg sin $45^0 - f_s$

$= mg\frac{1}{\sqrt{2}} - \mu N$,(as static frictional force, $f_s = \mu N$)

$= \frac{mg}{\sqrt{2}} - \mu mg\cos 45^\circ = mg\left[\frac{1}{\sqrt{2}} - \frac{\mu}{\sqrt{2}}\right]$ (as, normal force, N = mg cos 45°)

$\therefore ma = \frac{mg}{\sqrt{2}}[1-\mu] \Rightarrow a = \frac{g}{\sqrt{2}}(1-\mu)$

In this case time t = pT, distance $s = \frac{gT^2}{2\sqrt{2}}$ and acceleration $a = \frac{g}{\sqrt{2}}(1-\mu)$, again applying Newton's equation of motion,

$s = ut + \frac{1}{2}at^2 = 0 + \frac{1}{2}\cdot\frac{g}{\sqrt{2}}(1-\mu)(pT)^2$

$\therefore s = \frac{g}{2\sqrt{2}}(1-\mu)p^2T^2$ ...(ii)

Distances in both cases are equal (given). Hence, equating equation (i) with equation (ii) we get,

$\frac{gT^2}{2\sqrt{2}} = \frac{g}{2\sqrt{2}}(1-\mu)p^2T^2$

$\Rightarrow 1 = (1-\mu)p^2 \Rightarrow 1 = p^2 - \mu p^2$

$\Rightarrow \mu p^2 = p^2 - 1$

$\therefore \mu = \left[1 - \frac{1}{p^2}\right]$