Two similar kind of problems based on the concept of independent term.

Q1. Find the term independent of x, where x≠0, in the expansion of $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^{15}$

Sol. Given expansion is $\left(\frac{3x^2}{2} - \frac{1}{3x}\right)^{15}$

$$\therefore \quad T_{r+1} = {}^{15}C_r \left(\frac{3x^2}{2}\right)^{15-r} \left(-\frac{1}{3x}\right)^r$$

or $$T_{r+1} = {}^{15}C_r (-1)^r 3^{15-2r} 2^{r-15} x^{30-3r} \quad \text{(i)}$$

For the term independent of x, $30 - 3r = 0 \Rightarrow r = 10$

$\therefore$ The term independent of x is

$$T_{10+1} = {}^{15}C_{10}\, 3^{-5}\, 2^{-5} \quad \text{(Putting } r = 10 \text{ in (i))}$$

$$= {}^{15}C_{10}\left(\frac{1}{6}\right)^5$$

Q2. If the term free from x is the expansion of $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$ is 405, then find the value of k.

Sol: Given expansion is $\left(\sqrt{x} - \frac{k}{x^2}\right)^{10}$

$$\therefore \quad T_{r+1} = {}^{10}C_r (\sqrt{x})^{10-r} \left(\frac{-k}{x^2}\right)^r = {}^{10}C_r (x)^{\frac{1}{2}(10-r)} (-k)^r x^{-2r}$$

$$= {}^{10}C_r (x)^{5-\frac{r}{2}-2r} (-k)^r = {}^{10}C_r x^{\frac{10-5r}{2}} (-k)^r$$

For the term free from x, $\frac{10-5r}{2} = 0 \Rightarrow r = 2$

So, the term free from x is $T_{2+1} = {}^{10}C_2 (-k)^2$.

$\Rightarrow \quad {}^{10}C_2 (-k)^2 = 405$

$\Rightarrow \quad \frac{10 \times 9 \times 8!}{2! \times 8!}(-k)^2 = 405$

$\Rightarrow \quad 45k^2 = 405 \Rightarrow k^2 = 9 \quad \therefore k = \pm 3$