

Trick here is to just do exactly the thing that is being asked in question.

Example 15 If a_1, a_2, a_3 and a_4 are the coefficient of any four consecutive terms in the expansion of $(1+x)^n$, prove that

$$\frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{2a_2}{a_2 + a_3}$$

Solution Let a_1, a_2, a_3 and a_4 be the coefficient of four consecutive terms $T_{r+1}, T_{r+2}, T_{r+3}$, and T_{r+4} respectively. Then

$$\begin{aligned} a_1 &= \text{coefficient of } T_{r+1} = {}^n C_r \\ a_2 &= \text{coefficient of } T_{r+2} = {}^n C_{r+1} \\ a_3 &= \text{coefficient of } T_{r+3} = {}^n C_{r+2} \\ a_4 &= \text{coefficient of } T_{r+4} = {}^n C_{r+3} \end{aligned}$$

and

$$\begin{aligned} \text{Thus } \frac{a_1}{a_1 + a_2} &= \frac{{}^n C_r}{{}^n C_r + {}^n C_{r+1}} \\ &= \frac{{}^n C_r}{{}^{n+1} C_{r+1}} \quad (\because {}^n C_r + {}^n C_{r+1} = {}^{n+1} C_{r+1}) \\ &= \frac{|n|}{|r| |n-r|} \times \frac{|r+1| |n-r|}{|n+1|} = \frac{r+1}{n+1} \end{aligned}$$

Similarly,

$$\begin{aligned} \frac{a_3}{a_3 + a_4} &= \frac{{}^n C_{r+2}}{{}^n C_{r+2} + {}^n C_{r+3}} \\ &= \frac{{}^n C_{r+2}}{{}^{n+1} C_{r+3}} = \frac{r+3}{n+1} \end{aligned}$$

Hence,

$$\text{L.H.S.} = \frac{a_1}{a_1 + a_2} + \frac{a_3}{a_3 + a_4} = \frac{r+1}{n+1} + \frac{r+3}{n+1} = \frac{2r+4}{n+1}$$

and

$$\begin{aligned} \text{R.H.S.} &= \frac{2a_2}{a_2 + a_3} = \frac{2({}^n C_{r+1})}{{}^n C_{r+1} + {}^n C_{r+2}} = \frac{2({}^n C_{r+1})}{{}^{n+1} C_{r+2}} \\ &= 2 \frac{|n|}{|r+1| |n-r-1|} \times \frac{|r+2| |n-r-1|}{|n+1|} = \frac{2(r+2)}{n+1} \end{aligned}$$