QUES 05:-

Consider a cycle tyre being filled with air by a pump. Let *V* be the volume of the tyre (fixed) and at each stroke of the pump $\Delta V[<< V]$ of air is transferred to the tube adiabatically. What is the work done when the pressure in the tube is increased from P_1 to P_2?

Sol. Air is filled in the tyre adiabatically. let initial volume of air in tyre is V and after pumping one stroke volume become (V + dV) and pressure changes from P to (P + dP)

By adiabatic equation $P_1V_1^{\gamma} = P_2V_2^{\gamma}$

$P(V+dV)^{\gamma} = (P+dP)V^{\gamma}$

$PV^{\gamma}\left[1+\frac{dV}{V}\right]^{\gamma} = P\left[1+\frac{dP}{P}\right]V^{\gamma}$ **----(1)**

As volume of tyre V remains constant so dV/V is very small. By using binomial expansion in equation (1) we get

$PV^{\gamma}\left[1+\gamma\frac{dV}{V}\right] = PV^{\gamma}\left[1+\frac{dP}{P}\right]$

$1+\gamma\frac{dV}{V} = 1+\frac{dP}{P}$

on solving we get $dV = \frac{VdP}{\gamma P}$

Integrating both sidesnd using limits W_1 to W_2 for work done and P_1 to P_2 for pressure we get

$\int pdV = \int_{P_1}^{P_2} \frac{VdP}{\gamma}$

$\int_{W_1}^{W_2} dW = \frac{V}{\gamma}(P_2 - P_1)$(V)

$W_2 - W_1 = \frac{(P_2-P_1)V}{\gamma}$