

3. *Prove that* $|\mathbf{a} \times \mathbf{b}|^2 = a^2 b^2 \sin^2 \theta$

Solution Let $|\mathbf{a}| = a$, $|\mathbf{b}| = b$ and θ be the angle between \mathbf{a} and \mathbf{b} .

$$\begin{aligned} |\mathbf{a} \times \mathbf{b}|^2 &= (ab \sin \theta)^2 = a^2 b^2 \sin^2 \theta \\ &= a^2 b^2 (1 - \cos^2 \theta) = \\ &= a^2 b^2 - (\mathbf{a} \cdot \mathbf{b})^2 \end{aligned}$$