Three particles P, Q and R are moving along the vectors $\vec{A} = \hat{i} + \hat{j}, \vec{B} = \hat{j} + \hat{k}$ and $\vec{C} = -\hat{i} + \hat{j}$ respectively. They strike on a point and start to move in different directions. Now particle P is moving normal to the plane which contains vector $\vec{A}$ and $\vec{B}$. Similarly particle Q is moving normal to the plane which contains vector $\vec{A}$ and $\vec{C}$. The angle between the direction of motion of P and Q is $\cos^{-1}\left(\frac{1}{\sqrt{x}}\right)$. Then the value of x is _____.

[NA, July 22, 2021 (II)]

(3) Direction of particle P,

$$\hat{v}_1 = \pm\frac{\vec{A}\times\vec{B}}{|\vec{A}\times\vec{B}|} = \pm\frac{\hat{i}-\hat{j}+\hat{k}}{\sqrt{3}}$$

Direction of particle Q,

$$\hat{v}_2 = \pm\frac{\vec{A}\times\vec{C}}{|\vec{A}\times\vec{C}|} = \pm\frac{2\hat{k}}{2} = \pm\hat{k}$$

Angle between $\hat{v}_1$ and $\hat{v}_2$

$$\frac{\hat{v}_1 . \hat{v}_2}{|\hat{v}_1||\hat{v}_2|} = \frac{\pm\frac{1}{\sqrt{3}}}{(1)(1)} = \pm\frac{1}{\sqrt{3}}$$

Hence the angle between the direction of motion of P and Q is $\cos^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\therefore$ value of x = 3