Let $\vec{A}=(\hat{i}+\hat{j})$ and $\vec{B}=(\hat{i}-\hat{j})$. The magnitude of a coplanar vector $\vec{C}$ such that $\vec{A}.\vec{C}=\vec{B}.\vec{C}=\vec{A}.\vec{B}$ is given by [Main Online April 16, 2018]

(a) $\sqrt{\frac{5}{9}}$ (b) $\sqrt{\frac{10}{9}}$ (c) $\sqrt{\frac{20}{9}}$ (d) $\sqrt{\frac{9}{12}}$

(a) If $\vec{C}=a\hat{i}+b\hat{j}$ then $\vec{A}.\vec{C}=\vec{A}.\vec{B}$

$a+b=1$ ..... (i)

$\vec{B}.\vec{C}=\vec{A}.\vec{B}$

$2a-b=1$ ..... (ii)

Solving equation (i) and (ii) we get

$a=\frac{1}{3}, b=\frac{2}{3}$

$\therefore$ Magnitude of coplanar vector, $|\vec{C}|=\sqrt{\frac{1}{9}+\frac{4}{9}}=\sqrt{\frac{5}{9}}$