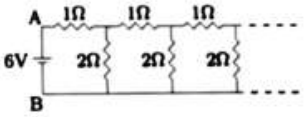
Q 13

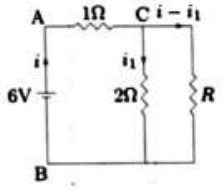
An infinite ladder network of resistances is constructed with 1 Ω and 2 Ω resistances (see figure). The 6 V battery between A and B has negligible internal resistance. (1987)



(a) Show that the effective resistance between A and B is 2 Ω.

(b) What is the current that passes through the 2 Ω resistance nearest to the battery?

Sol. Let the effective resistance between A and B be R. The ladder network consists of infinite number of units, where each unit consists of two resistances of values $R_1 = 1\ \Omega$ and $R_2 = 2\ \Omega$. The effective resistance of the ladder will not change by removal of one unit, say the unit close to the battery.



Thus, the effective resistance between A and B is equal to the resistance of the circuit shown in the figure i.e.,

$$R = R_1 + (R_2 \parallel R)$$
$$= R_1 + \frac{R_2 R}{R_2 + R} = 1 + \frac{2R}{2+R}.$$

Solve to get $R = 2\ \Omega$.

The effective resistance between A and B is $R = 2\ \Omega$. Thus, the current through the battery of *emf* $E =$ 6 V is $i = E/R = 6/2 = 3$ A. Since $R_2 = R = 2\ \Omega$, current i is equally divided at the node C giving $i_1 = i/2 = 1.5$ A.

Ans. (b) 1.5 A ⊡