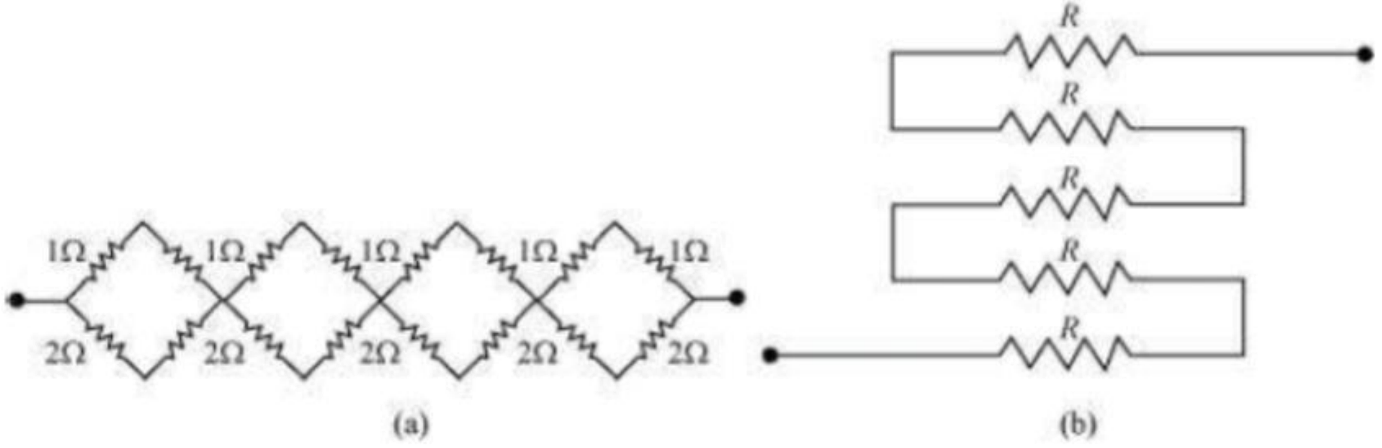
Q. 03

(a) Given *n* resistors each of resistance *R*, how will you combine them to get the (i) maximum (ii) minimum effective resistance? What is the ratio of the maximum to minimum resistance?

(b) Given the resistances of 1 Ω, 2 Ω, 3 Ω, how will be combine them to get an equivalent resistance of (i) (11/3) Ω (ii) (11/5) Ω, (iii) 6 Ω, (iv) (6/11) Ω?

(c) Determine the equivalent resistance of networks shown in Fig. 3.31.



Answer

(a) Total number of resistors = *n*

Resistance of each resistor = *R*

(i) When n resistors are connected in series, effective resistance R_1 is the maximum, given by the product nR.

Hence, maximum resistance of the combination, $R_1 = nR$

(ii) When n resistors are connected in parallel, the effective resistance (R_2) is the minimum, given by the ratio $\frac{R}{n}$.

Hence, minimum resistance of the combination, $R_2 = \frac{R}{n}$

(iii) The ratio of the maximum to the minimum resistance is,

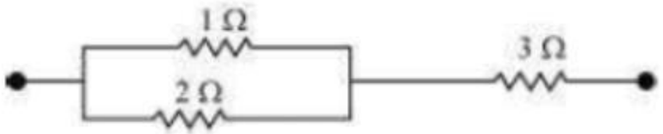
$$\frac{R_1}{R_2} = \frac{nR}{\frac{R}{n}} = n^2$$

(b) The resistance of the given resistors is,

$R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 3\ \Omega 2$

i. Equivalent resistance, $R' = \frac{11}{3}\ \Omega$

Consider the following combination of the resistors.

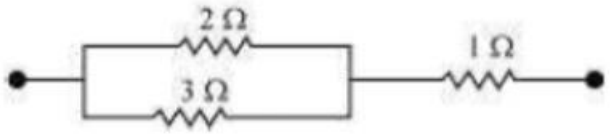


Equivalent resistance of the circuit is given by,

$$R' = \frac{2\times 1}{2+1} + 3 = \frac{2}{3} + 3 = \frac{11}{3}\ \Omega$$

ii. Equivalent resistance, $R' = \frac{11}{5}\ \Omega$

Consider the following combination of the resistors.



Equivalent resistance of the circuit is given by,

$$R' = \frac{2\times 3}{2+3} + 1 = \frac{6}{5} + 1 = \frac{11}{5}\ \Omega$$

(iii) Equivalent resistance, $R' = 6\ \Omega$

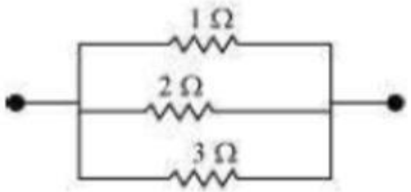
Consider the series combination of the resistors, as shown in the given circuit.

Equivalent resistance of the circuit is given by the sum,

$R' = 1 + 2 + 3 = 6\ \Omega$

(iv) Equivalent resistance, $R' = \frac{6}{11}\ \Omega$

Consider the series combination of the resistors, as shown in the given circuit.



Equivalent resistance of the circuit is given by,

$$R' = \frac{1\times 2\times 3}{1\times + 2\times 3 + 3\times 1} = \frac{6}{11}\ \Omega$$

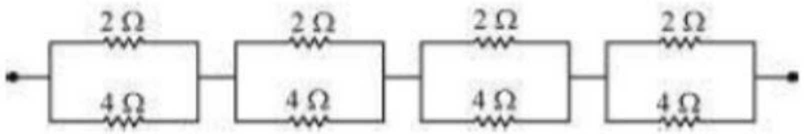
(c) (a) It can be observed from the given circuit that in the first small loop, two resistors of resistance 1 Ω each are connected in series.

Hence, their equivalent resistance = (1+1) = 2 Ω

It can also be observed that two resistors of resistance 2 Ω each are connected in series.

Hence, their equivalent resistance = (2 + 2) = 4 Ω.

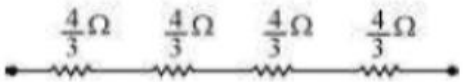
Therefore, the circuit can be redrawn as



It can be observed that 2 Ω and 4 Ω resistors are connected in parallel in all the four loops. Hence, equivalent resistance (R') of each loop is given by,

$$R' = \frac{2\times 4}{2+4} = \frac{8}{6} = \frac{4}{3}\ \Omega$$

The circuit reduces to,



All the four resistors are connected in series.

Hence, equivalent resistance of the given circuit is $\frac{4}{3}\times 4 = \frac{16}{3}\ \Omega$

(b) It can be observed from the given circuit that five resistors of resistance R each are connected in series.

Hence, equivalent resistance of the circuit = $R + R + R + R + R$

= 5 R

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