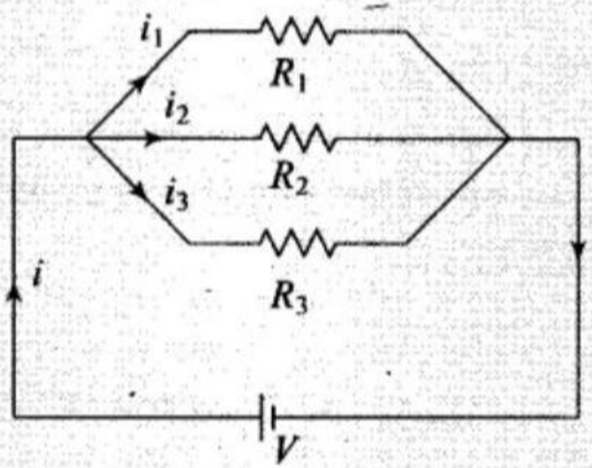
Q. 02

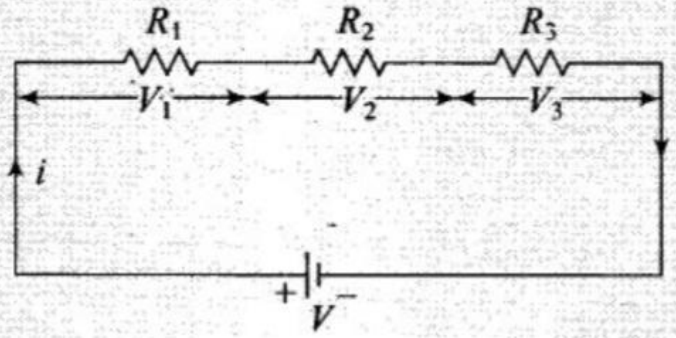
Let there be n resistors $R_1, \ldots, R_n$ with $R_{max} = \max(R_1, \ldots, R_n)$ and $R_{min} = \min\{R_1, \ldots, R_n\}$. Show that when they are connected in parallel, the resultant resistance $R_p = R_{min}$ and when they are connected in series, the resultant resistance $R_s > R_{max}$. Interpret the result physically.

Solution:

Key concept: *Parallel grouping*: Same potential difference appeared across each resistance but current distributes in the reverse ratio of their resistance, i.e. $i \propto \frac{1}{R}$



Series grouping: Same current flows through each resistance but potential difference distributes in the ratio of resistance, i.e. $V \propto R$



In parallel combination: When all resistances are connected in parallel, the equivalent resistance R_p is given by

$$\frac{1}{R_p} = \frac{1}{R_1} + \ldots + \frac{1}{R_n}$$

On multiplying both sides by $R_{\min}$, we have

$$\frac{R_{\min}}{R_p} = \frac{R_{\min}}{R_1} + \frac{R_{\min}}{R_2} + \ldots + \frac{R_{\min}}{R_n}$$

Here, in RHS, there exist one term $\frac{R_{\min}}{R_{\min}} = 1$ and other terms are positive, so we have

$$\frac{R_{\min}}{R_p} = \frac{R_{\min}}{R_1} + \frac{R_{\min}}{R_2} + \ldots + \frac{R_{\min}}{R_n} > 1$$

This shows that the resultant resistance $R_p < R_{\min}$.

Thus, in parallel combination, the equivalent resistance of resistors even less than the minimum resistance available in combination of resistors.

In series combination: When all resistances are connected in series, the equivalent resistance R_s is given by

$$R_s = R_1 + \ldots + R_n$$

Here, in RHS, there exist one term having resistance $R_{\max}$.

So, we have

or $\quad R_s = R_1 + \ldots + R_{\max} \ldots + \ldots + R_n$

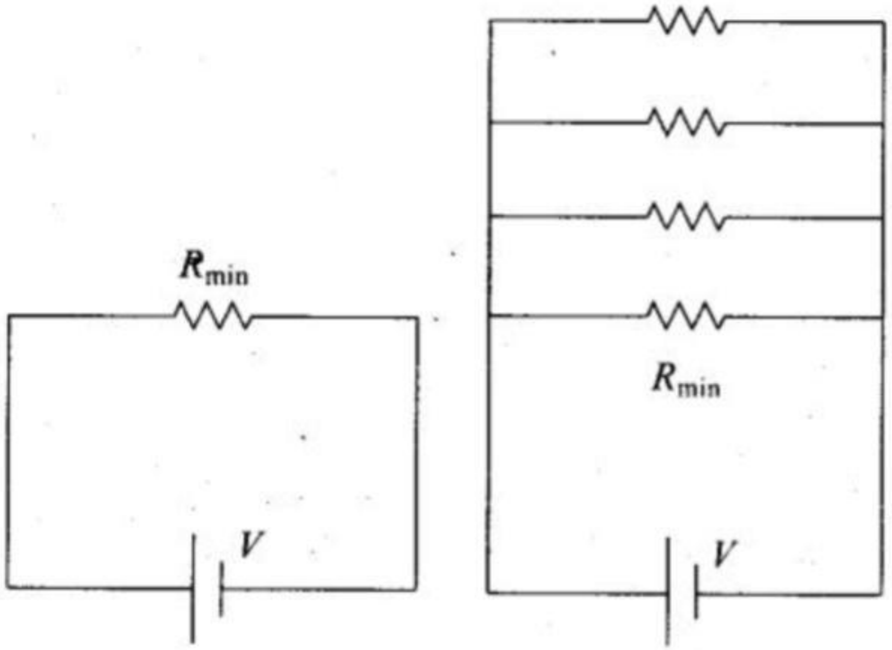
$$R_s = R_1 + \ldots + R_{\max} \ldots + R_n = R_{\max} + \ldots (R_1 + \ldots +)R_n$$

or $\quad R_s \geq R_{max}$

$$R_s = R_{max}(R_1 + \ldots + R_n)$$

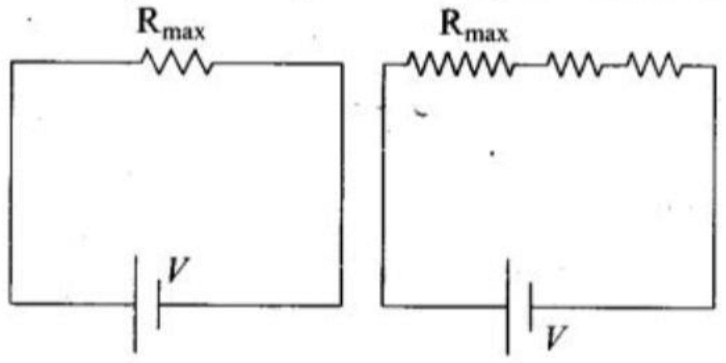
Thus, in series combination, the equivalent resistance of resistors is greater than the maximum resistance available in combination of resistors.

Physical interpretation:



(a) (b)

In Fig. (b), R_{min} provides an equivalent route as in Fig. (a) for current. But in addition there are $(n - 1)$ routes by the remaining $(n - 1)$ resistors. Current in Fig. (b) is greater than current in Fig. (a). Effective resistance in Fig. (b) $< R_{min}$. Second circuit evidently affords a greater resistance.



(c) (d)

In Fig. (d), R_{max} provides an equivalent route as in Fig. (c) for current. Current in Fig. (d) < current in Fig. (c). Effective resistance in Fig. (d) $> R_{max}$. Second circuit evidently affords a greater resistance.