Q. 05

Three vectors A,B and C add up to zero. Find which is false.

i. $(A\times B)\times C$ is not zero unless B,C are parallel
ii. $(A\times B).C$ is not zero unless B,C are parallel
iii. If A,B,C define a plane, $(A\times B)\times C$ is in that plane
iv. $(A\times B).C = |A|\,|B|\,|C| \rightarrow C^2 = A^2 + B^2$

Sol.

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We have to identify a false statement from above

We have given that $\vec{A}+\vec{B}+\vec{C} = 0$

Therefore taking the cross product on both sides

$\vec{A}+\vec{B}+\vec{C} = 0$

$\vec{B}\times(\vec{A}+\vec{B}+\vec{C}) = \vec{B}\times\vec{0}$

$\vec{B}\times\vec{A}+\vec{B}\times\vec{B}+\vec{B}\times\vec{C} = \vec{0}$

Now we know that when vectors are parallel then their cross product is zero. $\vec{B}\times\vec{B} = 0$

$\vec{A}\times\vec{B} = \vec{B}\times\vec{C}$

Taking post Cross Product on both sides with c

$(\vec{A}\times\vec{B})\times\vec{C} = (\vec{B}\times\vec{C})\times\vec{C}$

Now this could only zero when B and C are parallel to each other as

$\vec{B}\times\vec{C} = |B||C|\sin\theta = 0$ only when $\theta = 0$ that's when B and C are parallel

Therefore statement A is true.

Now taking the previous equation

$\vec{A}\times\vec{B} = \vec{B}\times\vec{C}$

Taking dot product with c on both sides

$(\vec{A}\times\vec{B})\cdot\vec{C} = (\vec{B}\times\vec{C})\cdot\vec{C}$

Now this could be zero on two conditions first is that B and C are parall
without C being parallel to B. As when we will take the cross product of
perpendicular to both B and C, say vector K. And by taking the dot prod
zero as the angle between them will always be 90.

Therefore B is false

Now if vector triple product of A and B and C, then vector will always lie
formed by A, B and C. This could be visualized by understanding that $\vec{A}$
lie in a single plane forming sides of the triangle.

Now,

$\vec{A}\times\vec{B} = \vec{K}$

K will be perpendicular to the plane containing A and B.

And taking the cross product with C (which is also lying on the same pla
give a vector which is perpendicular to C but will be lying on the same p

Therefore statement C is true.

It is given in last option that $|\vec{A}\times\vec{B}|$ = |A| |B|, therefore, the angle betw
B is 90 and we know that

$\vec{A}+\vec{B}+\vec{C}$ = 0, therefore |A|, |B|, |C| form a triangle with the angle bet
therefore, it is a right-angled triangle.

Hence option D is also true.