

4) If the coefficients of x^2 and x^3 are both zero, in the expansion of the expression $(1+ax+bx^2)(1-3x)^{15}$ in powers of x , then the ordered pair (a, b) is equal to:

[Main April 10, 2019 (1)]

(a) (28, 861) (b) (-54, 315)

(c) (28, 315) (d) (-21, 714)

Solution: (c) Given expression is $(1+ax+bx^2)(1-3x)^{15}$

Co-efficient of $x^2 = 0$

$$\Rightarrow {}^{15}C_2(-3)^2 + a \cdot {}^{15}C_1(-3) + b \cdot {}^{15}C_0 = 0$$

$$\Rightarrow \frac{15 \times 14}{2} \times 9 - 15 \times 3a + b = 0$$

$$\Rightarrow 945 - 45a + b = 0 \quad \text{--- (1)}$$

Now, co-efficient of $x^3 = 0$

$$\Rightarrow {}^{15}C_3(-3)^3 + a \cdot {}^{15}C_2(-3)^2 + b \cdot {}^{15}C_1(-3) = 0$$

$$\Rightarrow \frac{15 \times 14 \times 13}{3 \times 2} \times (-3 \times 3 \times 3) + a \times \frac{15 \times 14 \times 9}{2} - b \times 3 \times 15 = 0$$

$$\Rightarrow 15 \times 3 [-3 \times 7 \times 13 + a \times 7 \times 3 - b] = 0$$

$$\Rightarrow 21a - b = 273 \quad \text{--- (2)}$$

From (1) and (2), we get,

$$a = 28, b = 315 \Rightarrow (a, b) \equiv \boxed{28, 315}$$