

3) The coefficient of t^4 in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$:

- (a) 14 (b) 15 (c) 10 (d) 12

[Main Jan 09, 2019 (II)]

Solution: (b)

Consider the expression

$$\left(\frac{1-t^6}{1-t}\right)^3 = (1-t^6)(1-t)^{-3}$$

$$= (1-3t^6+3t^{12}-t^{18}) \left(1+3t+\frac{3\cdot 4}{2!}t^2+\frac{3\cdot 4\cdot 5}{3!}t^3+\right.$$

$$\left.+\frac{3\cdot 4\cdot 5\cdot 6}{4!}t^4+\dots+\infty\right)$$

Hence the coefficient of t^4

$$= \frac{1 \cdot 3 \cdot 4 \cdot 5 \cdot 6}{4!}$$

$$= \frac{3 \times 4 \times 5 \times 6}{4 \times 3 \times 2 \times 1} = \boxed{15}$$