1) The value of

$$({}^{21}C_1 - {}^{10}C_1) + ({}^{21}C_2 - {}^{10}C_2) + ({}^{21}C_3 - {}^{10}C_3) + ({}^{21}C_4 - {}^{10}C_4) + \ldots + ({}^{21}C_{10} - {}^{10}C_{10})$$

is:

(a) $2^{20} - 2^{10}$ (b) $2^{21} - 2^{11}$ [Main 2017]

(c) $2^{21} - 2^{10}$ (d) $2^{20} - 2^{9}$

Solution: (a)

We have $({}^{21}C_1 + {}^{21}C_2 + \ldots + {}^{21}C_{10}) - ({}^{10}C_1 + {}^{10}C_2 + \ldots {}^{10}C_{10})$

$$= \frac{1}{2}\left[({}^{21}C_1 + \ldots + {}^{21}C_{10}) + {}^{21}C_{11} + \ldots {}^{21}C_{20})\right] - (2^{10} - 1)$$

$$= \frac{1}{2}\left[2^{21} - 2\right] - (2^{10} - 1) = (2^{20} - 1) - (2^{10} - 1) = \boxed{2^{20} - 2^{10}}$$