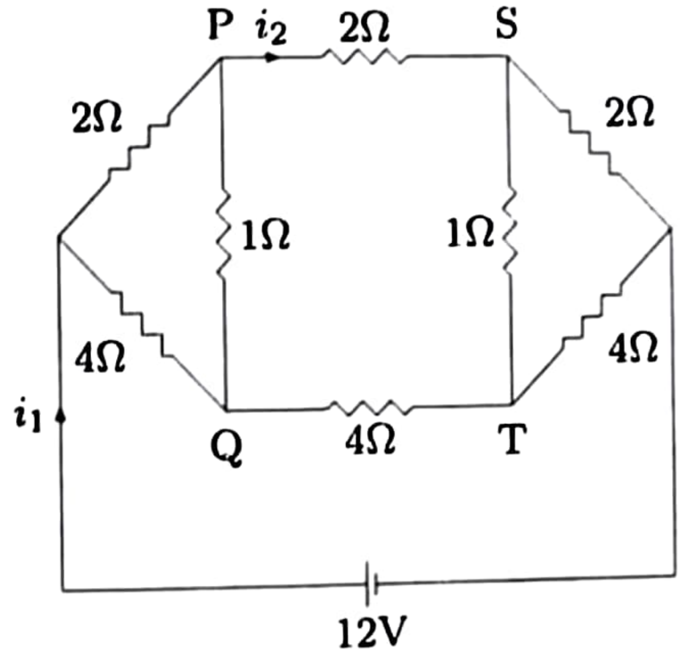
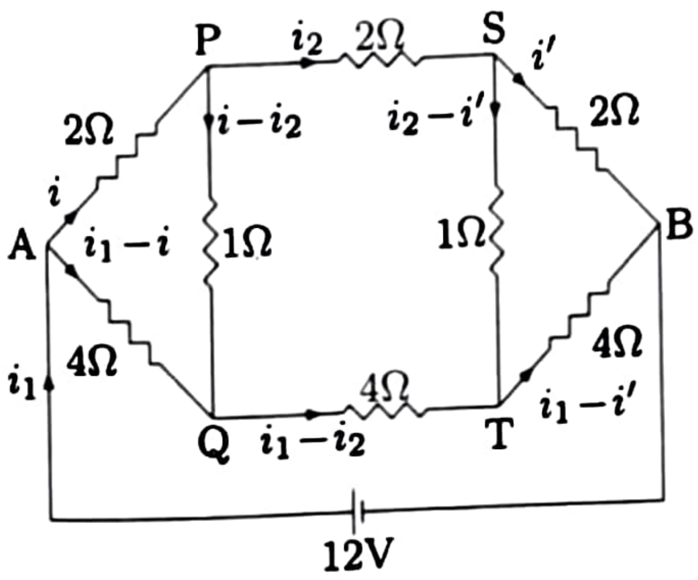
Q. 12 For the resistance network shown in the figure, choose the correct option(s), (2012)



(A) The current through PQ is zero.
(B) $i_1 = 3$ A
(C) The potential at S is less than that at Q.
(D) $i_2 = 2$ A

Sol. Let i be the current leaving node A through branch AP and i' be the current entering node B through branch SB. Kirchhoff's junction law gives the currents in other branches.



We claim, by symmetry, that $i = i'$ because circuit looks symmetrical when we see it from either node A or B. This can be shown by applying Kirchhoff's loop law in loops APQA and SBTS which gives

$$2i + (i - i_2) - 4(i_1 - i) = 0, \tag{1}$$

$$2i' - 4(i_1 - i') - (i_2 - i') = 0. \tag{2}$$

Solve equations (1) and (2) to get

$$i = i' = (4i_1 + i_2)/7. \tag{3}$$

Again, by using symmetry, currents in branch PQ and ST are zero. This can be shown by applying Kirchhoff's loop law in PSTQP which gives

$$2i_2 + (i_2 - i') - 4(i_1 - i_2) - (i - i_2) = 0. \tag{4}$$

Substitute i and i' from equation (3) into equation (4) and simplify to get $i_1 = \frac{3}{2}i_2$, and $i = i_2$. Now, apply Kirchhoff's loop law in APSBA (through battery) to get, $6i_2 = 12$. Solve to get, $i_2 = i = i' = 2$ A and $i_1 = 3$ A. Taking the path QAPS, the potential at S is $V_S = V_Q + 4(i_1 - i) - 2i - 2i_2 = V_Q - 4$ V. It may be noted that the problem can be solved by using Kirchhoff's laws only without utilizing symmetry arguments. The readers are encouraged to put the conditions which break the symmetry and solve the problem.

Ans. A, B, C, D ⊡