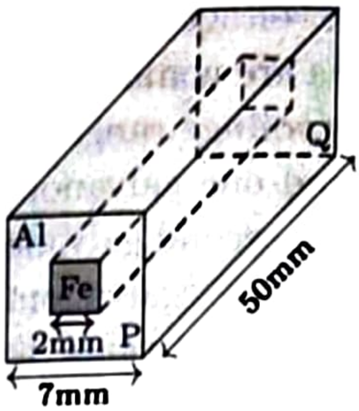
Q. 11 In an aluminium (Al) bar of square cross-section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are $2.7 \times 10^{-8}\ \Omega$ m and $1.0 \times 10^{-7}\ \Omega$ m, respectively. The electrical resistance between the two faces P and Q of the composite bar is (2015)



(A) $\frac{2475}{64}\ \mu\Omega$ (B) $\frac{1875}{64}\ \mu\Omega$ (C) $\frac{1875}{49}\ \mu\Omega$ (D) $\frac{2475}{132}\ \mu\Omega$

Sol. The resistance of a conductor of length l, cross-sectional area A, and material resistivity ρ is given by $R = \rho l/A$. For iron bar, length is $l_{\text{Fe}} = 5 \times 10^{-2}$ m, cross-sectional area is $A_{\text{Fe}} = (2\times10^{-3})^2 = 4 \times 10^{-6}$ m^2, and resistivity is $\rho_{\text{Fe}} = 1.0 \times 10^{-7}\Omega$ m.

The resistance of the iron bar is

$$R_{\text{Fe}} = \frac{\rho_{\text{Fe}} l_{\text{Fe}}}{A_{\text{Fe}}} = \frac{(1 \times 10^{-7})(5 \times 10^{-2})}{4 \times 10^{-6}} = 1250\ \mu\Omega.$$

For the aluminium bar, $A_{\text{Al}} = (7 \times 10^{-3})^2 - (2 \times 10^{-3})^2 = 4.5 \times 10^{-5}$ m^2, $l_{\text{Al}} = 5 \times 10^{-2}$ m, and $\rho_{\text{Al}} = 2.7 \times 10^{-8}\Omega$ m. The resistance of the aluminium bar is

$$R_{\text{Al}} = \frac{\rho_{\text{Al}} l_{\text{Al}}}{A_{\text{Al}}} = \frac{(2.7 \times 10^{-8})(5 \times 10^{-2})}{4.5 \times 10^{-5}} = 30\ \mu\Omega.$$

The iron and aluminium bars are connected in parallel between P and Q. Thus, the equivalent resistance of the composite bar is

$$R_{\text{eq}} = R_{\text{Fe}} \parallel R_{\text{Al}} = \frac{R_{\text{Fe}} R_{\text{Al}}}{R_{\text{Fe}} + R_{\text{Al}}} = \frac{(1250)(30)}{1250 + 30}$$

$$= \frac{1875}{64}\ \mu\Omega.$$

Ans. B ☐