Q. 04

Two wires of equal length, one of aluminium and the other of copper have the same resistance. Which of the two wires is lighter? Hence explain why aluminium wires are preferred for overhead power cables. ($\rho_{Al} = 2.63 \times 10^{-8}\ \Omega$ m, $\rho_{Cu} = 1.72 \times 10^{-8}\ \Omega$ m, Relative density of Al = 2.7, of Cu = 8.9.)

Answer

Resistivity of aluminium, $\rho_{Al} = 2.63 \times 10^{-8}\ \Omega$ m

Relative density of aluminium, $d_1 = 2.7$

Let l_1 be the length of aluminium wire and m_1 be its mass.

Resistance of the aluminium wire = R_1

Area of cross-section of the aluminium wire = A_1

Resistivity of copper, $\rho_{Cu} = 1.72 \times 10^{-8}\ \Omega$ m

Relative density of copper, $d_2 = 8.9$

Let l_2 be the length of copper wire and m_2 be its mass.

Resistance of the copper wire = R_2

Area of cross-section of the copper wire = A_2

The two relations can be written as

$$R_1 = \rho_1 \frac{l_1}{A_1} \quad \ldots (1)$$

$$R_2 = \rho_2 \frac{l_2}{A_2} \quad \ldots (2)$$

It is given that,

$R_1 = R_2$

$$\rho_1 \frac{l_1}{A_1} = \rho_2 \frac{l_2}{A_2}$$

And,

$l_1 = l_2$

$\therefore \frac{\rho_1}{A_1} = \frac{\rho_2}{A_2}$

$\frac{A_1}{A_2} = \frac{\rho_1}{\rho_2}$

$= \frac{2.63 \times 10^{-8}}{1.72 \times 10^{-8}} = \frac{2.63}{1.72}$

Mass of the aluminium wire,

m_1 = Volume × Density

$= A_1 l_1 \times d_1 = A_1 \, l_1 d_1$ ... (3)

Mass of the copper wire,

m_2 = Volume × Density

$= A_2 l_2 \times d_2 = A_2 \, l_2 d_2$ ... (4)

Dividing equation (3) by equation (4), we obtain

$\frac{m_1}{m_2} = \frac{A_1 l_1 d_1}{A_2 l_2 d_2}$

For $l_1 = l_2$,

$\frac{m_1}{m_2} = \frac{A_1 d_1}{A_2 d_2}$

For $\frac{A_1}{A_2} = \frac{2.63}{1.72}$,

$$\frac{m_1}{m_2} = \frac{2.63}{1.72} \times \frac{2.7}{8.9} = 0.46$$

It can be inferred from this ratio that m_1 is less than m_2. Hence, aluminium is lighter than copper.

Since aluminium is lighter, it is preferred for overhead power cables over copper.