Concepts and Formulas

1. **Injection Principle(IP):**

Let there be two sets A and B such that they are finite, meaning they have finite elements. If there exists a one-to-one mapping f from A to B, then

$$n(A) <= n(B)$$

2. **Bijection Principle(BP):**

Let there be two sets A and B such that they are finite, meaning they have finite elements. If there exists a bijection mapping f between A and B, then

$$n(A) = n(B)$$

3. **Fundamental theorem of arithmetic:**

A composite number is expressed in the form of the product of primes and this factorization is unique apart from the order in which the prime factor occurs.

statement: a composite number "a" can be expressed as, $a = p1\ p2\ p3 \ldots\ pn$, where $p1, p2, p3 \ldots\ pn$ are the prime factors of a written in ascending order i.e. $p1 \le p2 \le p3 \ldots \le pn$.

4. Now combine FTA and bijection principle to get no. of divisors of any number. See examples in video and problem pdfs.

5. **The Inclusion-Exclusion Principle:**

Suppose two tasks A and B can be performed simultaneously. Let n(A) and n(B) represent the number of ways of performing the tasks A and B independent of each other. The principle says:

$$n(A \cup B) = n(A) + n(B) - n(A \cap B).$$

6. **Occupancy Problems:**

The occupancy problem in probability theory is about the problem of randomly assigning a set of balls into a group of cells.

Result-1: the number of ways of distributing r identical balls in n distinct boxes is given by:

$$\binom{n+r-1}{n-1} = \binom{n+r-1}{r}$$

Result-2: the number of ways of distributing r identical balls in n distinct boxes so that no boxes remains empty, is given by:

$$\binom{r-1}{n-1} = \binom{r-1}{r-n} \quad \text{Here} \quad r \geq n$$

Result-3: the number of ways of distributing r distinct balls in n distinct boxes so that each box hold at least one ball, is given by:

$$^{n}P_{r} \quad \text{here} \quad r \leq n$$

Result-4: the number of ways of distributing r distinct balls in n distinct boxes so that any box hold any number of balls, is given by:

$$n^r$$

Result-5: the number of ways of distributing r distinct balls in n distinct boxes such that ordering of balls matters in each box, is given by:

$$\binom{n+r-1}{r} r!$$

Try to understand proof of these results from video lecture.