Vectors - Class XII

Related Questions with Solutions

Questions

Quetion: 01

If $\vec{a} = \hat{i} + \hat{j} + \hat{k}, \vec{b} = 4\hat{i} + 3\hat{j} + 4\hat{k}$ and $\vec{c} = \hat{i} + \alpha\hat{j} + \beta\hat{k}$ are linearly dependent vectors and $|\vec{c}| = \sqrt{3}$, then

A. $\alpha = 1, \beta = -1$

B. $\alpha = 1, \beta = \pm 1$

C. $\alpha = -1, \beta = \pm 1$

D. $\alpha = \pm 1, \beta = 1$

Solutions

Solution: 01

If $\vec{a}, \vec{b}, \vec{c}$ are linearly dependent vectors, then $\vec{c}$ should be a linear combination of $\vec{a}$ and $\vec{b}$.

Let $\vec{c} = p\vec{a} + q\vec{b}$

i.e., $\hat{i} + \alpha\hat{j} + \beta\hat{k} = p(\hat{i} + \hat{j} + \hat{k}) + q(4\hat{i} + 3\hat{j} + 4\hat{k})$

Equating coefficients of $\hat{i}, \hat{j}, \hat{k}$ from both sides, we get

$1 = p + 4q, \alpha = p + 3q, \beta = p + 4q$

From first and third, $\beta = 1$

Now, $|\vec{c}| = \sqrt{3}[\text{ Given }] \Rightarrow 1 + \alpha^2 + \beta^2 = 3$

$\Rightarrow 1 + \alpha 2 + 1 = 3 \quad \Rightarrow \quad \alpha = \pm 1$

Hence, $\alpha = \pm 1, \beta = 1$

Correct Options

Answer:01

Correct Options: D