Concepts and Formulas

Dot Product of Two Vectors:

$$\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta$$

Properties:

(i) $\vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$ [i.e. dot product is commutative].

(ii) $(\vec{a}\cdot\vec{b})\cdot\vec{c}$ is not defined.

(iii) $\vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b}+\vec{a}\cdot\vec{c}$ [distributive property]

(iv) If $\vec{a}$ and $\vec{b}$ are perpendicular to each other, then $\vec{a}\cdot\vec{b} = 0$, converse is also true.

(v) Projection of $\vec{a}$ on $\vec{b} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{b}|}$ and projection of $\vec{b}$ on $\vec{a} = \dfrac{\vec{a}\cdot\vec{b}}{|\vec{a}|}$.

(vi) If $\theta = 0$, then the projection vector of $\overrightarrow{AB}$ will be $\overrightarrow{AB}$ itself and if $\theta = \pi$, then the projection vector of $\overrightarrow{AB}$ will be $\overrightarrow{BA}$.

(vii) If $\theta = \dfrac{\pi}{2}$ or $\theta = \dfrac{3\pi}{2}$, then the projection vector of $\overrightarrow{AB}$ will be zero vector.

(viii) Angle between two vectors $\vec{a}$ and $\vec{b}$ is

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|} \quad \text{or} \quad \theta = \cos^{-1}\left[\frac{\vec{a}\cdot\vec{b}}{|\vec{a}|\cdot|\vec{b}|}\right]$$

(ix) $\vec{a}\cdot\vec{a} = |\vec{a}|^2$

(x) $\hat{i}^2 = \hat{j}^2 = \hat{k}^2 = 1$

(xi) $\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0$

(xii) If $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, then $\vec{a}\cdot\vec{b} = a_1b_1+a_2b_2+a_3b_3$.

(xiii) $(\lambda\cdot\vec{a})\cdot\vec{b} = \lambda(\vec{a}\cdot\vec{b}) = \vec{a}\cdot(\lambda\cdot\vec{b})$, where λ is any scalar.

(xiv) If $\theta = \pi$, then $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|$; If $\theta = \pi$, then $\vec{a}\cdot\vec{b} = -|\vec{a}||\vec{b}|$

Vector (or Cross) Product of Vectors:

$$\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|\sin\theta\,\hat{n}, \text{ such that } 0\le\theta\le\pi$$

here n makes right handed system with two vectors a and b.

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Properties:

(i) Angle between two vectors is $\sin\theta = \frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}$ or $\theta = \sin^{-1}\left[\frac{|\vec{a}\times\vec{b}|}{|\vec{a}||\vec{b}|}\right]$

(ii) $\vec{a}\times\vec{a} = 0$

(iii) $\vec{a}\times\vec{b} = -\vec{b}\times\vec{a}$

(iv) In general, $\vec{a}\times(\vec{b}\times\vec{c}) \neq (\vec{a}\times\vec{b})\times\vec{c}$

(v) $\vec{a}\times(\vec{b}+\vec{c}) = \vec{a}\times\vec{b}+\vec{a}\times\vec{c}$ [distributive property]

(vi) $\lambda(\vec{a}\times\vec{b}) = (\lambda\vec{a})\times\vec{b} = \vec{a}\times(\lambda\vec{b})$

(vii) If $\vec{a}$ is parallel to $\vec{b}$, then $\vec{a}\times\vec{b} = \vec{0}$ and converse is also true.

(viii) If $\theta = \frac{\pi}{2}$, then $\vec{a}\times\vec{b} = |\vec{a}||\vec{b}|$.

(ix) Area of parallelogram whose adjacent sides are along $\vec{a}$ and $\vec{b} = |\vec{a}\times\vec{b}|$.

(x) Area of triangle, whose adjacent sides are along $\vec{a}$ and $\vec{b} = \frac{1}{2}|\vec{a}\times\vec{b}|$.

(xi) $\hat{i}\times\hat{i} = \hat{j}\times\hat{j} = \hat{k}\times\hat{k} = \vec{0}$ and $\hat{i}\times\hat{j} = \hat{k}, \hat{j}\times\hat{k} = \hat{i}, \hat{k}\times\hat{i} = \hat{j}$

(xii) $\hat{j}\times\hat{i} = -\hat{k},\ \hat{k}\times\hat{j} = -\hat{i}$ and $\hat{i}\times\hat{k} = -\hat{j}$

(xiii) If $\vec{a} = a_1\hat{i}+a_2\hat{j}+a_3\hat{k}$ and $\vec{b} = b_1\hat{i}+b_2\hat{j}+b_3\hat{k}$, then $\vec{a}\times\vec{b} = \begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ a_1 & a_2 & a_3\\ b_1 & b_2 & b_3\end{vmatrix}$

$\Rightarrow \quad (a_2b_3 - a_3b_2)\hat{i} + (a_3b_1 - a_1b_3)\hat{j} + (a_1b_2 - a_2b_1)\hat{k}$

(xiv) Unit vector $\hat{n}$, which is perpendicular to both the vectors $\vec{a}$ and $\vec{b}$, is given by

$$\hat{n} = \frac{\vec{a}\times\vec{b}}{|\vec{a}\times\vec{b}|}$$

(xv) For vectors $\vec{a}$ and $\vec{b}$, if $\vec{a}\times\vec{b} = \vec{0}$, then either $\vec{a} = \vec{0}$ or $\vec{b} = \vec{0}$ or $\vec{a}||\vec{b}$.