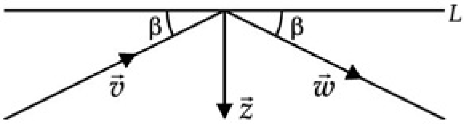
Vectors - Class XII

Related Questions with Solutions

Questions

Quetion: 01

The figure shows non-zero vectors $\vec{v}, \vec{w}$ and $\vec{z}$ with $\vec{z}$ orthogonal to the line L, and $\vec{v}$ and $\vec{w}$ making equal angles β with the line L. Assuming $|\vec{v}|=|\vec{w}|$, if the vector $\vec{w}$ is expressed as a linear combination of $\vec{v}$ and $\vec{z}$ as $\vec{w}=x\vec{v}+y\vec{z}$ then



A. x = 1

B. $x=\dfrac{v\sin\beta}{z}$

C. y = 2

D. $y=\dfrac{2v\sin\beta}{z}$

Solutions

Solution: 01

Let, $|\vec{v}|=|\vec{w}|=v, |\vec{z}|=z$

Given $\vec{w}=x\vec{v}+y\vec{z}$

$\vec{w}\cdot\vec{v}=|\vec{w}|\cdot|\vec{v}|\cos 2\beta=v^2\cos 2\beta$

$\vec{w}\cdot\vec{z}=|\vec{w}|\cdot|\vec{z}|\cos\left(\frac{\pi}{2}-\beta\right)=vz\sin\beta$

$\vec{v}\cdot\vec{z}=|\vec{v}||\vec{z}|\cos\left(\frac{\pi}{2}+\beta\right)=-vz\sin\beta$

Now $\vec{w}=x\vec{v}+y\vec{z}$

Taking dot product with $\vec{v}$, we get

$\vec{w}\cdot\vec{v}=x\vec{v}\cdot\vec{v}+y\vec{z}\cdot\vec{v}$

$\Rightarrow v^2\cos 2\beta=v^2x+y(-vz\sin\beta)$

$\Rightarrow (v)x-(z\sin\beta)y=v\cos 2\beta$ ....[i]

Taking dot product with $\vec{w}$, we get

$\vec{w}\cdot\vec{w}=x\vec{v}\cdot\vec{w}+y\vec{z}\cdot\vec{w}$

$\Rightarrow v^2=xv^2\cos 2\beta+yvz\sin\beta$

$\Rightarrow (v\cos 2\beta)x+(z\sin\beta)y=v$ .... [ii]

Solving equation [i] and [ii], we have

$x=1$ and $y=\dfrac{2v\sin\beta}{z}$

Correct Options

Answer:01

Correct Options: A, D