

Vectors - Class XII

Past Year JEE Questions

Questions

Question: 01

Let $\vec{a} = \hat{i} - \alpha\hat{j} + \beta\hat{k}$, $\vec{b} = 3\hat{i} + \beta\hat{j} - \alpha\hat{k}$ and $\vec{c} = \alpha\hat{i} - 2\hat{j} + \hat{k}$, where α and β are integers. If $\vec{a} \cdot \vec{b} = -1$ and $\vec{b} \cdot \vec{c} = 10$, then $(\vec{a} \times \vec{b}) \cdot \vec{c}$ is equal to _____.

Solutions

Solution: 01

Answer

Correct Answer is **9**

Explanation

$$\vec{a} = (1, -\alpha, \beta)$$

$$\vec{b} = (3, \beta, -\alpha)$$

$$\vec{c} = (-\alpha, -2, 1); \alpha, \beta \in I$$

$$\vec{a} \cdot \vec{b} = -1 \Rightarrow 3 - \alpha\beta - \alpha\beta = -1$$

$$\Rightarrow \alpha\beta = 2$$

Possible value of
 α and β

$$\begin{array}{cc} 1 & 2 \\ 2 & 1 \\ -1 & -2 \\ -2 & -1 \end{array}$$

$$\vec{b} \cdot \vec{c} = 10$$

$$\Rightarrow -3\alpha - 2\beta - \alpha = 10$$

$$\Rightarrow 2\alpha + \beta + 5 = 0$$

$$\therefore \alpha = -2; \beta = -1$$

$$[\vec{a} \ \vec{b} \ \vec{c}] = \begin{vmatrix} 1 & 2 & -1 \\ 3 & -1 & 2 \\ 2 & -2 & 1 \end{vmatrix}$$

$$= 1(-1 + 4) - 2(3 - 4) - 1(-6 + 2)$$

$$= 3 + 2 + 4 = 9$$