

Vectors - Class XII

Past Year JEE Questions

Questions

Question: 01

If $(\vec{a} + 3\vec{b})$ is perpendicular to $(7\vec{a} - 5\vec{b})$ and $(\vec{a} - 4\vec{b})$ is perpendicular to $(7\vec{a} - 2\vec{b})$, then the angle between \vec{a} and \vec{b} (in degrees) is _____.

Solutions

Solution: 01

Answer

Correct Answer is **60**

Explanation

$$(\vec{a} + 3\vec{b}) \perp (7\vec{a} - 5\vec{b})$$

$$\therefore (\vec{a} + 3\vec{b}) \cdot (7\vec{a} - 5\vec{b}) = 0$$

$$\Rightarrow 7|\vec{a}|^2 - 15|\vec{b}|^2 + 16\vec{a} \cdot \vec{b} = 0 \dots(1)$$

$$\text{Also, } (\vec{a} - 4\vec{b}) \cdot (7\vec{a} - 2\vec{b}) = 0$$

$$\Rightarrow 7|\vec{a}|^2 + 8|\vec{b}|^2 - 30\vec{a} \cdot \vec{b} = 0 \dots(2)$$

Equation (1) \times 30

$$210|\vec{a}|^2 - 450|\vec{b}|^2 + 480\vec{a} \cdot \vec{b} = 0 \dots(3)$$

Equation (2) \times 16

$$112|\vec{a}|^2 + 128|\vec{b}|^2 - 480\vec{a} \cdot \vec{b} = 0 \dots(4)$$

from (3) & (4)

$$332|\vec{a}|^2 = 332|\vec{b}|^2$$

$$\Rightarrow |\vec{a}|^2 = |\vec{b}|^2$$

$$\Rightarrow |\vec{a}| = |\vec{b}|$$

From equation (2),

$$15 |\vec{a}| = 30 \vec{a} \cdot \vec{b}$$

$$\Rightarrow 15 |\vec{a}|^2 = 30 |\vec{a}| \cdot |\vec{b}| \cos \theta$$

$$\cos \theta = \frac{15}{30} = \frac{1}{2}$$

$$\therefore \theta = 60^\circ$$