V_{AB} = 2 A × 2 Ω = 4 V

The voltage drop across BC is

V_{BC} = 2 A × 1 Ω = 2 V

Finally, the voltage drop across CD is

$V_{CD} = 12\ \Omega \times I_3 = 12\ \Omega \times \left(\frac{2}{3}\right) \text{A} = 8\ \text{V}.$

This can alternately be obtained by multiplying total current between C and D by the equivalent resistance between C and D, that is,

V_{CD} = 2 A × 4 Ω = 8 V

Note that the total voltage drop across AD is 4 V + 2 V + 8 V = 14 V. Thus, the terminal voltage of the battery is 14 V, while its emf is 16 V. The loss of the voltage (= 2 V) is accounted for by the internal resistance 1 Ω of the battery [2 A × 1 Ω = 2 V].

EXAMPLE 3.5

3.12 Cells in Series and in Parallel

Like resistors, cells can be combined together in an electric circuit. And like resistors, one can, for calculating currents and voltages in a circuit, replace a combination of cells by an equivalent cell.

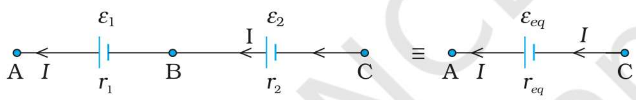

FIGURE 3.20 Two cells of emf's ε_1 and ε_2 in the series. r_1, r_2 are their internal resistances. For connections across A and C, the combination can be considered as one cell of emf ε_{eq} and an internal resistance r_{eq}.

Consider first two cells in series (Fig. 3.20), where one terminal of the two cells is joined together leaving the other terminal in either cell free. ε_1, ε_2 are the emf's of the two cells and r_1, r_2 their internal resistances, respectively.

Let V(A), V(B), V(C) be the potentials at points A, B and C shown in Fig. 3.20. Then V(A) – V(B) is the potential difference between the positive and negative terminals of the first cell. We have already calculated it in Eq. (3.57) and hence,

$$V_{\text{AB}} \equiv V(\text{A}) - V(\text{B}) = \varepsilon_1 - I r_1 \tag{3.60}$$

Similarly,

$$V_{\text{BC}} \equiv V(\text{B}) - V(\text{C}) = \varepsilon_2 - I r_2 \tag{3.61}$$

Hence, the potential difference between the terminals A and C of the combination is

$$V_{\text{AC}} \equiv V(\text{A}) - V(\text{C}) = \left[V(\text{A}) - V(\text{B})\right] + \left[V(\text{B}) - V(\text{C})\right]$$

$$= (\varepsilon_1 + \varepsilon_2) - I(r_1 + r_2) \tag{3.62}$$

If we wish to replace the combination by a single cell between A and C of emf ε_{eq} and internal resistance r_{eq}, we would have

$$V_{AC} = \varepsilon_{eq} - I\, r_{eq} \tag{3.63}$$

Comparing the last two equations, we get

$$\varepsilon_{eq} = \varepsilon_1 + \varepsilon_2 \tag{3.64}$$

and $r_{eq} = r_1 + r_2$ (3.65)

In Fig.3.20, we had connected the negative electrode of the first to the positive electrode of the second. If instead we connect the two negatives, Eq. (3.61) would change to $V_{BC} = -\varepsilon_2 - Ir_2$ and we will get

$$\varepsilon_{eq} = \varepsilon_1 - \varepsilon_2 \qquad (\varepsilon_1 > \varepsilon_2) \tag{3.66}$$

The rule for series combination clearly can be extended to any number of cells:

(i) The equivalent emf of a series combination of n cells is just the sum of their individual emf's, and

(ii) The equivalent internal resistance of a series combination of n cells is just the sum of their internal resistances.

This is so, when the current leaves each cell from the positive electrode. If in the combination, the current leaves any cell from the *negative* electrode, the emf of the cell enters the expression for ε_{eq} with a *negative* sign, as in Eq. (3.66).

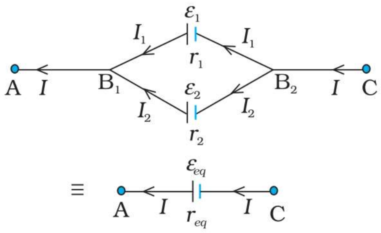

FIGURE 3.21 Two cells in parallel. For connections across A and C, the combination can be replaced by one cell of emf ε_{eq} and internal resistances r_{eq} whose values are given in Eqs. (3.73) and (3.74).

Next, consider a parallel combination of the cells (Fig. 3.21). I_1 and I_2 are the currents leaving the positive electrodes of the cells. At the point B_1, I_1 and I_2 flow in whereas the current I flows out. Since as much charge flows in as out, we have

$$I = I_1 + I_2 \tag{3.67}$$

Let $V(B_1)$ and $V(B_2)$ be the potentials at B_1 and B_2, respectively. Then, considering the first cell, the potential difference across its terminals is $V(B_1) - V(B_2)$. Hence, from Eq. (3.57)

$$V \equiv V(B_1) - V(B_2) = \varepsilon_1 - I_1 r_1 \tag{3.68}$$

Points B_1 and B_2 are connected exactly similarly to the second cell. Hence considering the second cell, we also have

$$V \equiv V(B_1) - V(B_2) = \varepsilon_2 - I_2 r_2 \tag{3.69}$$

Combining the last three equations

$$I = I_1 + I_2$$

$$= \frac{\varepsilon_1 - V}{r_1} + \frac{\varepsilon_2 - V}{r_2} = \left(\frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2}\right) - V\left(\frac{1}{r_1} + \frac{1}{r_2}\right) \tag{3.70}$$

Hence, V is given by,

$$V = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} - I\frac{r_1 r_2}{r_1 + r_2} \tag{3.71}$$

If we want to replace the combination by a single cell, between B_1 and B_2, of emf ε_{eq} and internal resistance r_{eq}, we would have

$$V = \varepsilon_{eq} - I\, r_{eq} \tag{3.72}$$

The last two equations should be the same and hence

$$\varepsilon_{eq} = \frac{\varepsilon_1 r_2 + \varepsilon_2 r_1}{r_1 + r_2} \tag{3.73}$$

$$r_{eq} = \frac{r_1 r_2}{r_1 + r_2} \tag{3.74}$$

We can put these equations in a simpler way,

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \frac{1}{r_2} \tag{3.75}$$

$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \frac{\varepsilon_2}{r_2} \tag{3.76}$$

In Fig. (3.21), we had joined the positive terminals together and similarly the two negative ones, so that the currents I_1, I_2 flow out of positive terminals. If the negative terminal of the second is connected to positive terminal of the first, Eqs. (3.75) and (3.76) would still be valid with $\varepsilon_2 \rightarrow -\varepsilon_2$

Equations (3.75) and (3.76) can be extended easily. If there are n cells of emf ε_1, . . . ε_n and of internal resistances r_1,... r_n respectively, connected in parallel, the combination is equivalent to a single cell of emf ε_{eq} and internal resistance r_{eq}, such that

$$\frac{1}{r_{eq}} = \frac{1}{r_1} + \ldots + \frac{1}{r_n} \tag{3.77}$$

$$\frac{\varepsilon_{eq}}{r_{eq}} = \frac{\varepsilon_1}{r_1} + \ldots + \frac{\varepsilon_n}{r_n} \tag{3.78}$$

GUSTAV ROBERT KIRCHHOFF (1824 – 1887)

Gustav Robert Kirchhoff (1824 – 1887) German physicist, professor at Heidelberg and at Berlin. Mainly known for his development of spectroscopy, he also made many important contributions to mathematical physics, among them, his first and second rules for circuits.

3.13 Kirchhoff's Rules

Electric circuits generally consist of a number of resistors and cells interconnected sometimes in a complicated way. The formulae we have derived earlier for series and parallel combinations of resistors are not always sufficient to determine all the currents and potential differences in the circuit. Two rules, called *Kirchhoff's rules*, are very useful for analysis of electric circuits.

Given a circuit, we start by labelling currents in each resistor by a symbol, say *I*, and a directed arrow to indicate that a current *I* flows along the resistor in the direction indicated. If ultimately *I* is determined to be positive, the actual current in the resistor is in the direction of the arrow. If *I* turns out to be negative, the current actually flows in a direction opposite to the arrow. Similarly, for each source (i.e., cell or some other source of electrical power) the positive and negative electrodes are labelled, as well as, a directed arrow with a symbol for the current flowing through the cell. This will tell us the potential difference, $V = V(P) - V(N) = \varepsilon - I r$