may be hundreds of miles away, via transmission cables. One obviously wants to minimise the power loss in the transmission cables connecting the power stations to homes and factories. We shall see now how this can be achieved. Consider a device R, to which a power P is to be delivered via transmission cables having a resistance R_c to be dissipated by it finally. If V is the voltage across R and I the current through it, then

$$P = V\,I \tag{3.34}$$

The connecting wires from the power station to the device has a finite resistance R_c. The power dissipated in the connecting wires, which is wasted is P_c with

$$P_c = I^2 R_c$$
$$= \frac{P^2 R_c}{V^2} \tag{3.35}$$

from Eq. (3.32). Thus, to drive a device of power P, the power wasted in the connecting wires is inversely proportional to V^2. The transmission cables from power stations are hundreds of miles long and their resistance R_c is considerable. To reduce P_c, these wires carry current at enormous values of V and this is the reason for the high voltage danger signs on transmission lines — a common sight as we move away from populated areas. Using electricity at such voltages is not safe and hence at the other end, a device called a transformer lowers the voltage to a value suitable for use.

3.10 Combination of Resistors – Series and Parallel

The current through a single resistor R across which there is a potential difference V is given by Ohm's law $I = V/R$. Resistors are sometimes joined together and there are simple rules for calculation of equivalent resistance of such combination.

FIGURE 3.13 A series combination of two resistors R_1 and R_2.

Two resistors are said to be in *series* if only one of their end points is joined (Fig. 3.13). If a third resistor is joined with the series combination of the two (Fig. 3.14), then all three are said to be in series. Clearly, we can extend this definition to series combination of any number of resistors.

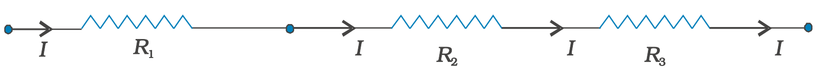

FIGURE 3.14 A series combination of three resistors R_1, R_2, R_3.

Two or more resistors are said to be in *parallel* if one end of all the resistors is joined together and similarly the other ends joined together (Fig. 3.15).

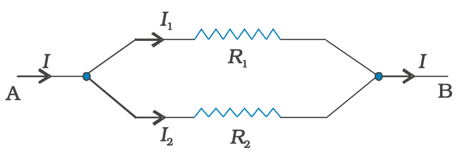

FIGURE 3.15 Two resistors R_1 and R_2 connected in parallel.

Consider two resistors R_1 and R_2 in series. The charge which leaves R_1 must be entering R_2. Since current measures the rate of flow of charge, this means that the same current I flows through R_1 and R_2. By Ohm's law:

Potential difference across $R_1 = V_1 = I R_1$, and

Potential difference across $R_2 = V_2 = I R_2$.

The potential difference V across the combination is V_1+V_2. Hence,

$$V = V_1 + V_2 = I (R_1 + R_2) \tag{3.36}$$

This is as if the combination had an equivalent resistance R_{eq}, which by Ohm's law is

$$R_{eq} \equiv \frac{V}{I} = (R_1 + R_2) \tag{3.37}$$

If we had three resistors connected in series, then similarly

$$V = I R_1 + I R_2 + I R_3 = I (R_1 + R_2 + R_3). \tag{3.38}$$

This obviously can be extended to a series combination of any number n of resistors $R_1, R_2 \ldots\ldots, R_n$. The equivalent resistance R_{eq} is

$$R_{eq} = R_1 + R_2 + \ldots + R_n \tag{3.39}$$

Consider now the parallel combination of two resistors (Fig. 3.15). The charge that flows in at A from the left flows out partly through R_1 and partly through R_2. The currents I, I_1, I_2 shown in the figure are the rates of flow of charge at the points indicated. Hence,

$$I = I_1 + I_2 \tag{3.40}$$

The potential difference between A and B is given by the Ohm's law applied to R_1

$$V = I_1 R_1 \tag{3.41}$$

Also, Ohm's law applied to R_2 gives

$$V = I_2 R_2 \tag{3.42}$$

$$\therefore\ I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = V\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{3.43}$$

If the combination was replaced by an equivalent resistance R_{eq}, we would have, by Ohm's law

$$I = \frac{V}{R_{eq}} \tag{3.44}$$

Hence,

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} \tag{3.45}$$

We can easily see how this extends to three resistors in parallel (Fig. 3.16).

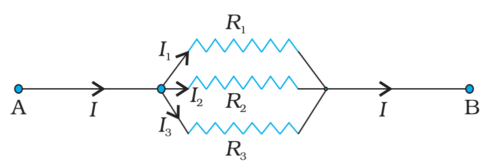

FIGURE 3.16 Parallel combination of three resistors R_1, R_2 and R_3.

Exactly as before

$I = I_1 + I_2 + I_3$ (3.46)

and applying Ohm's law to R_1, R_2 and R_3 we get,

$V = I_1 R_1,\ V = I_2 R_2,\ V = I_3 R_3$ (3.47)

So that

$$I = I_1 + I_2 + I_3 = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \quad (3.48)$$

An equivalent resistance R_{eq} that replaces the combination, would be such that

$$I = \frac{V}{R_{eq}} \quad (3.49)$$

and hence

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \quad (3.50)$$

We can reason similarly for any number of resistors in parallel. The equivalent resistance of n resistors $R_1, R_2 \ldots, R_n$ is

$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \ldots + \frac{1}{R_n} \quad (3.51)$$

These formulae for equivalent resistances can be used to find out currents and voltages in more complicated circuits. Consider for example, the circuit in Fig. (3.17), where there are three resistors R_1, R_2 and R_3. R_2 and R_3 are in parallel and hence we can replace them by an equivalent R_{eq}^{23} between point B and C with

$$\frac{1}{R_{eq}^{23}} = \frac{1}{R_2} + \frac{1}{R_3}$$

or, $$R_{eq}^{23} = \frac{R_2 R_3}{R_2 + R_3} \quad (3.52)$$

The circuit now has R_1 and R_{eq}^{23} in series and hence their combination can be replaced by an equivalent resistance with

$$R_{eq}^{123} = R_{eq}^{23} + R_1 \quad (3.53)$$

If the voltage between A and C is V, the current I is given by

$$I = \frac{V}{R_{eq}^{123}} = \frac{V}{R_1 + \left[R_2 R_3/(R_2 + R_3)\right]}$$

$$= \frac{V(R_2 + R_3)}{R_1 R_2 + R_1 R_3 + R_2 R_3} \quad (3.54)$$

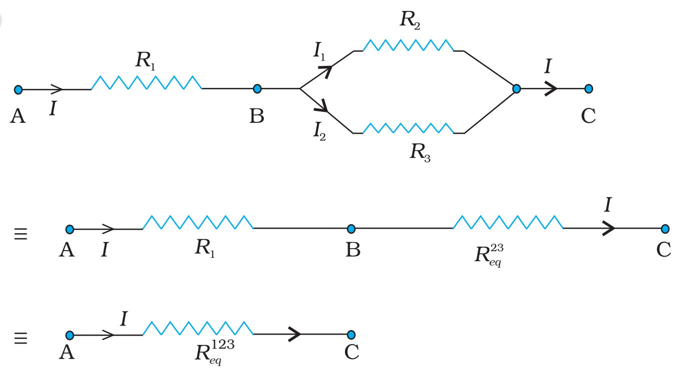

FIGURE 3.17 A combination of three resistors R_1, R_2 and R_3. R_2, R_3 are in parallel with an equivalent resistance R_{eq}^{23}. R_1 and R_{eq}^{23} are in series with an equivalent resistance R_{eq}^{123}.