

Q10. The solution set of the equation

$\log_x 2 \log_{2x} 2 = \log_{4x} 2$ is

- (a) $\{2^{-\sqrt{2}}, 2^{\sqrt{2}}\}$ (b) $\{1/2, 2\}$
(c) $\{1/4, 2^2\}$ (d) none of these

Ans 10

$$\begin{aligned}\therefore \quad & \log_x 2 \log_{2x} 2 = \log_{4x} 2 \\ \therefore \quad & x > 0, 2x \neq 0 \text{ and } 4x > 0 \text{ and} \\ & x \neq 1, 2x \neq 1, 4x \neq 1 \\ \Rightarrow \quad & x > 0 \text{ and } x \neq 1, \frac{1}{2}, \frac{1}{4}\end{aligned}$$

Then, $\frac{1}{\log_2 x} \cdot \frac{1}{\log_2 2x} = \frac{1}{\log_2 4x}$

$$\begin{aligned}\Rightarrow \quad & \log_2 x \cdot \log_2 2x = \log_2 4x \\ \Rightarrow \quad & \log_2 x \cdot (1 + \log_2 x) = (2 + \log_2 x) \\ \Rightarrow \quad & (\log_2 x)^2 = 2 \\ \Rightarrow \quad & \log_2 x = \pm \sqrt{2} \\ \therefore \quad & x = 2^{\pm \sqrt{2}} \\ \therefore \quad & x = \{2^{-\sqrt{2}}, 2^{\sqrt{2}}\}\end{aligned}$$

Hence, the correct option is (a).