

**Question 7: If  $\log_{10} 5 = a$  and  $\log_{10} 3 = b$  then**

(a)  $\log_{30} 8 = 3(1-a)/(b+1)$

(b)  $\log_{40} 15 = (a+b)/(3-2a)$

(c)  $\log_{243} 32 = (1-a)/b$

(d) none of these

**Solution:**

Given  $\log_{10} 5 = a$  and  $\log_{10} 3 = b$

We check all the given options.

$$\log_{30} 8 = \log 2^3 / \log (3 \times 10)$$

$$= 3 \log 2 / (\log 3 + \log 10)$$

$$= 3 \log (10/5) / (1 + \log 3)$$

$$= 3 (1 - \log 5) / (1 + \log 3)$$

$$= 3(1 - a) / (1 + b)$$

Hence option a is correct.

$$\log_{40} 15 = \log 15 / \log 40$$

$$= \log(3 \times 5) / \log(10 \times 4)$$

$$= (\log 3 + \log 5) / (1 + \log 2^2)$$

$$= (\log 3 + \log 5) / (1 + 2 \log 2)$$

$$= (\log 3 + \log 5) / (1 + 2 \log (10/5))$$

$$= (\log 3 + \log 5) / (1 + 2(1 - \log 5))$$

$$= (\log 3 + \log 5) / (1 + 2 - 2 \log 5)$$

$$= (b+a) / (3 - 2a)$$

Hence option b is correct.

$$\log_{243} 32 = \log 32 / \log 243$$

$$= \log 2^5 / \log 3^5$$

$$= 5 \log 2 / 5 \log 3$$

$$= \log 2 / \log 3$$

$$= (1 - \log 5) / \log 3 \text{ (since } \log 2 = \log (10/5) = \log 10 - \log 5 = 1 - \log 5)$$

$$= (1-a) / b$$

So option c is correct.

Hence option a, b and c are correct.