Question 3: The number of positive integers satisfying the equation $x + \log_{10}(2^x + 1) = x\log_{10}5 + \log_{10}6$ is

(a) 0

(b) 1

(c) 2

(d) infinite

Solution:

$x + \log_{10}(2^x + 1) = x\log_{10}5 + \log_{10}6$

$x[1 - \log_{10}5] + \log_{10}(2^x + 1) = \log_{10}6$

$x[\log_{10}10 - \log_{10}5] + \log_{10}(2^x + 1) = \log_{10}6$

$x\log_{10}2 + \log_{10}(2^x + 1) = \log_{10}6$

$\log_{10}2^x(2^x + 1) = \log_{10}6$

$(2^x)^2 + 2^x - 6 = 0$

$2^x = 2$ or $2^x = -3$ (rejected)

$\Rightarrow x = 1$

So number of positive integers = 1

Hence option b is the answer.