Question 1: Let (x_0, y_0) be the solution of the following equations.

$$(2x)^{\ln 2} = (3y)^{\ln 3}$$

$3^{\ln x} = 2^{\ln y}$. Then x_0 is

(a) 1/6

(b) 1/3

(c) 1/2

(d) 6

Solution:

Given that $(2x)^{\ln 2} = (3y)^{\ln 3}$ ..(i)

Taking log on both sides

$\log 2 \log (2x) = \log 3 \log (3y)$

$\log 2 (\log 2 + \log x) = \log 3 (\log 3 + \log y)$ ..(ii)

Also $3^{\ln x} = 2^{\ln y}$ ..(iii)

Taking log on both sides

$\log x \log 3 = \log y \log 2$

$\log y = \log x \log 3 / \log 2$ ...(iv)

Substitute (iv) in (ii)

$\log 2 (\log 2 + \log x) = \log 3 (\log 3 + \log x \log 3 / \log 2)$

$(\log 2)^2 + \log 2 \log x = (\log 3)^2 + \log x (\log 3)^2 / \log 2$

$[\log 2 - (\log 3)^2 / \log 2] \log x = (\log 3)^2 - (\log 2)^2$

$\log x = [(\log 3)^2 - (\log 2)^2] / [(\log 2)^2 - (\log 3)^2 / \log 2]$

$\log x = - \log 2$

$\log x = \log 2^{-1}$

$\Rightarrow x = 2^{-1}$

$= 1/2$

Hence option c is the answer.