

Example 28 Is it true that $x = e^{\log x}$ for all real x ?

Solution First, observe that the domain of log function is set of all positive real numbers. So the above equation is not true for non-positive real numbers. Now, let $y = e^{\log x}$. If $y > 0$, we may take logarithm which gives us $\log y = \log (e^{\log x}) = \log x \cdot \log e = \log x$. Thus $y = x$. Hence $x = e^{\log x}$ is true only for positive values of x .

One of the striking properties of the natural exponential function in differential calculus is that it doesn't change during the process of differentiation. This is captured in the following theorem whose proof we skip.